

Annex 3 - Demo reference point analysis for Western Mediterranean stock examples with FLRef

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1 Loading packages and data for MUT stocks

Load required packages

```
library(FLCore)
library(FLBRP)
library(FLasher)
library(FLSRTMB)
library(ggplotFL)
library(FLRef)
library(ggpubr) # For this demo
Warning: package 'ggpubr' was built under R version 4.1.3
```

This RMD file is located in the subfolder FLRef/Rcodes. The FLStocks objects for Western Mediterranean are loaded from FLRef/data.

```
load("../data/STEFc_Medstocks2021.Rdata", verbose = TRUE)
Loading objects:
  stks
```

Each FLStock contains a number of additional attributes, including species name, taxonomic order. For finfishes, it also contains meta-analysis estimates about the stock relationship that were extracted from the FishLife model by Thorson (2000)

```
stk = stks$MUT01
stk@name
[1] "MUT01"
stk@desc
[1] "STEFc,a4a,2021"
stk@species
[1] "Mullus barbatus barbatus"
stk@taxonomy
$species
[1] "Mullus barbatus barbatus"

$order
[1] "Perciformes"
stk@fishlife
      rho      sigmaR      s sdlogit.s
0.2580000 0.7977181 0.7730000 1.0563095
```

```
plot(stks) + theme_bw() + facet_wrap(~qname, scales = "free", ncol = 2)
```

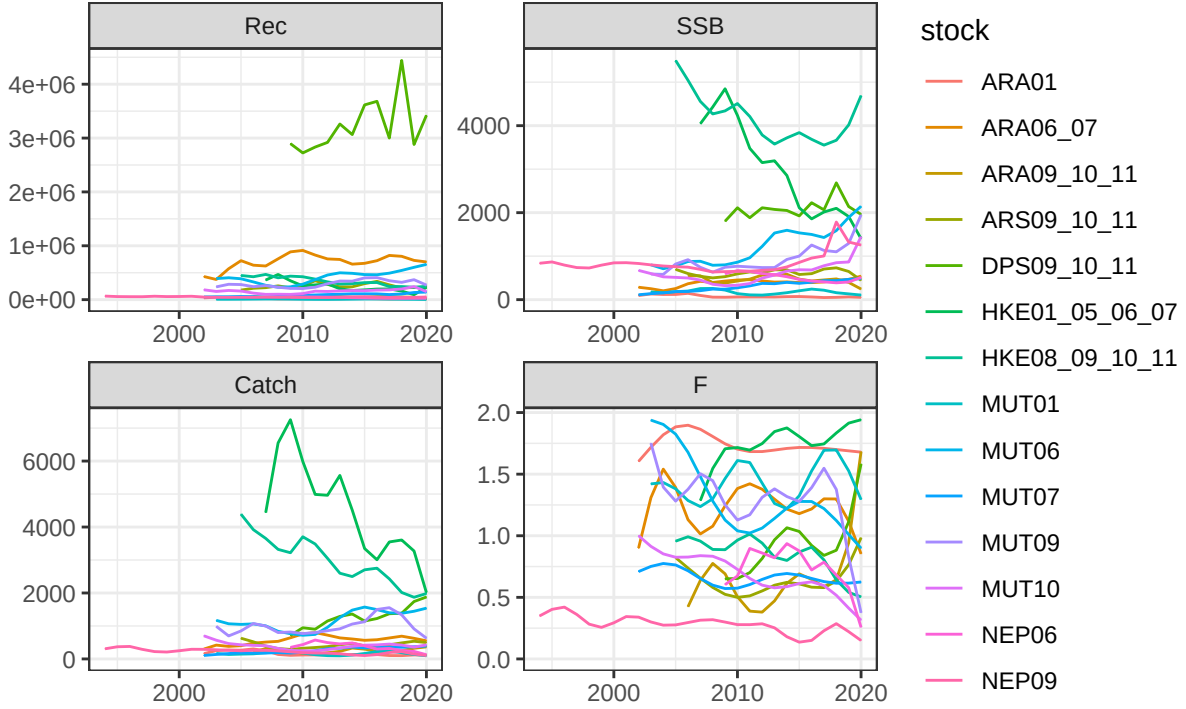


Figure 1: Stock assessment trajectories for Western Mediterranean stocks

2 General approach and Assumptions

2.1 Population biology

Biological reference points (*BRPs*) are inherently linked to the assumed stock biology in terms of the input quantities of weight at age (W_a), maturity at age (Mat_a) and natural mortality at age (M_a) as well the estimated selectivity-at-age (Sel_a), whereby also for Sel_a strong assumptions about the functional form are made a priori. Here is assumed the accepted assessment represents the best available information about the stock dynamics. Nevertheless, it critical to link the implications of the stock dynamics to the reference point estimates to enable adequate interpretation. In particular, implausibility of biomass reference points may quickly attributed difficulties of estimating the stock recruitment relationship (S-R), whereas much the seemingly unrealistic scaling is already implicit to the conflict between the biological assumption and the data (i.e. model specification).

As part of the toolbox of *FLRef*, a number plotting functions can be routinely run to support this evaluation:

- `plotdyn()`
- `plotbioyr()`
- `plotbioage()`

2.2 Per-recruit analysis

Per-recruit analysis provides a useful tool to evaluate the stock dynamics in terms the truncation of the age-structure trunction in terms “growth overfishing” in absence of a S-R. As a basic principle, the average spawning ratio potential $SRP = SPR_F / SPR_0$ is at least as high or higher than the average SSB_y / SBB_0

derived from an S-R, with the exception a strongly dome-shaped Ricker S-R. This is because the scale of SSB_0 is given by $SSB_0 = R_0 SPR_0$.

Here, the following assumptions are made for the per-recruit analysis:

- The F_{MSY} proxy is taken to be $F_{0.1}$
- Recruitment is constant in the absence of a S-R function
- $B_{F0.1}$ is the corresponding spawning biomass per-recruit (SPR)
- SPR_0 is the unfished SPR at $F = 0$
- Spawning Ratio Potential SRP denotes the SPR_F / SPR_0
- B_{lim} set to $0.25B_{F0.1}$ as a first approximation

2.3 Estimating stock recruitment (S-R) relationships

Stock recruitment relationships (SSR) can be fitted and evaluated with the **FLR** (Fisheries Library in R: Kell et al., 2007) package **FLSRTMB** (Winker and Mosqueira; <https://github.com/flr/FLSRTMB>), using maximum likelihood estimation in Templet Model Builder (TMB; Kristensen 2015) for the following stock-recruitment functions:

- A conditioned Hockey-Stick (**segreg**)
- Beverton & Holt (**bevholtSV**)
- Ricker (**rickerSV**)

In addition, the geometric recruitment (**geomean**) can be computed using a simple linear model with a single intercept term, such that:

$$\log(R_y) = a$$

where R_y is the number of recruits in year y and the intercept a determines expected mean recruitment $\hat{R} = \exp(a)$, which is in this case independent of SSB and therefore is equal to the unfished recruitment R_0 .

2.3.1 Conditioned Hockey-Stick

To estimate the break-point $b = B_{lim}$ within the expected range of 1% - 20% of SSB_0 , a novel formulation conditional Hockey-Stick formulation was implemented in **FLSRTMB**. The Hockey-Stick function is based on a continuous, quadratic hockey-stick formulation (c.f. Barrowman and Myers), which is re-parameterised as a function of the nonstationary SPR_{0_y} in year y and a “re-purposed” steepness parameter s^* . In addition, the parameter P_{lim} is introduced, which then determines the lower of the ratio B_{lim}/B_0 , where B_{lim} corresponds to break point b of the segmented regression and the unfished B_0 being a function of $B_0 = R_0 SPR_0$.

$$R_y = \frac{s^*}{2P_{lim}SPR_{0_y}} \left(SSB_{y-a_r} + P_{lim}R_0SPR_{0_y}/s^* - \sqrt{(SSB_{y-a_r} - P_{lim}R_0SPR_{0_y}/s^*)^2} \right)$$

where R_y is the number of recruits in year y , SSB_{y-a_r} is the spawning biomass in year y minus minimum age a_{min} defined for the stock (typically age-0 or age-1). The break point b (B_{lim}) and slope a are given by

$$b = P_{lim}R_0SPR_0/s^*$$

$$a = R_0/b$$

where SPR_0 can be set based on, e.g., the mean SPR_{y_0} over the time-series or mean SPR_{y_0} over the most recent years.

This formulation allows to bound the parameter s^* over an approximately uniform range defined for $0 > s^* \leq 1$ using a truncated logistic prior distribution (Thorson and Cope, 2014)

$$s \sim TrunkLogit(s_{logit}, \sigma_{logit})$$

where σ_{logit} is set by default to 20 and s^* can be truncated by specifying the parameter c .

$$s^* = c + 1/(1 + \exp(-s_{logit}))$$

where c is specified by ratio lower to upper fraction of SPR_0 . For example, by considering a range bounded between 1% - 20% of SPR_0 , c is taken a ratio of $c = 0.01/0.2$, resulting in an approximately uniform parameter space for which the ratio B_{lim}/B_0 , but with soft penalties towards the bounds to facilitate robust convergence.

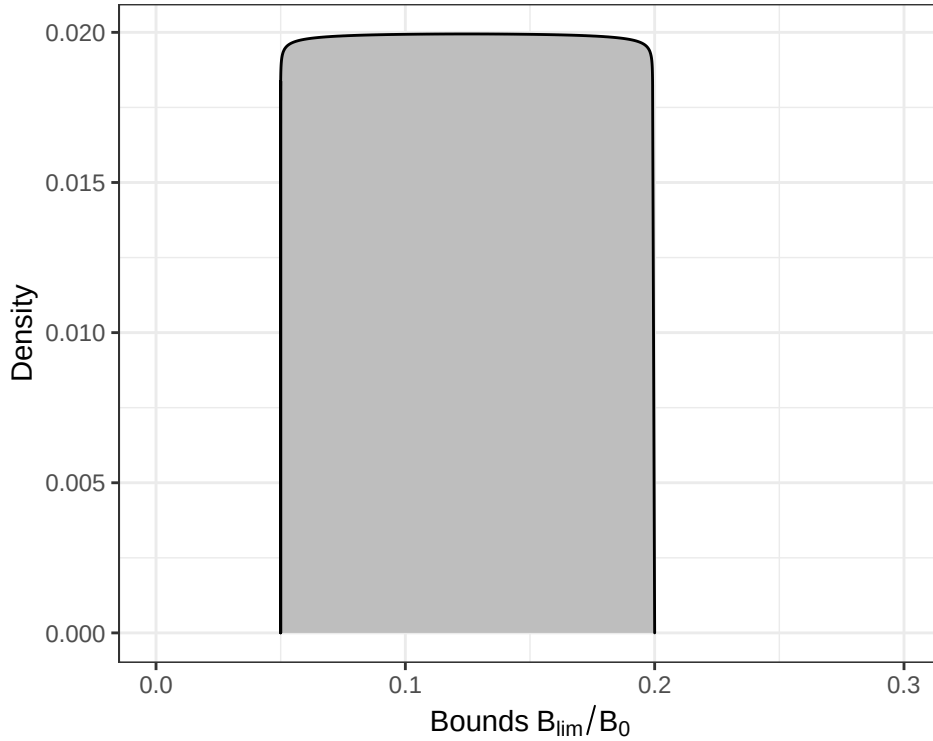


Figure 2: Defined parameter space for the relative location of break-point $b = B_{lim}$, shown for conditioning the hockey-stick model with bounds at 0.01 - 0.2 of SPR_0 .

The properties of the conditioning continuous hockey-stick model can be related as follows to the four ICES types of S-R patterns:

1. If a break-point is identifiable within the specified range of $0.01 - 0.2SPR_0$, the estimated break-point $b = B_{lim}$ is consistent with ICES Type 2.

2. If S-R data overlap with at least the range $0.1 - 0.2SPR_0$ and there is no positive relationship between SSB and recruitment, B_{lim} will be closely approximated $B_{loss} = SSB_{min}$ (Type 5) and the R_0 estimate is equal to the geometric mean recruitment.
3. If the S-R data overlap with the range $0.01 - 0.2SPR_0$ and there is positive relationship between SSB and recruitment, B_{lim} will typically be located closely to SSB_{min} (Type 3)
4. If the S-R data fall outside $0.2SPR_0$ or outside $0.1SPR_0$ with no identifiable break-point between $0.01 - 0.2SPR_0$, B_{lim} will be located just below $0.1SPR_0$ (i.e. mean of range) and the R_0 estimate is equal to the geometric mean recruitment.

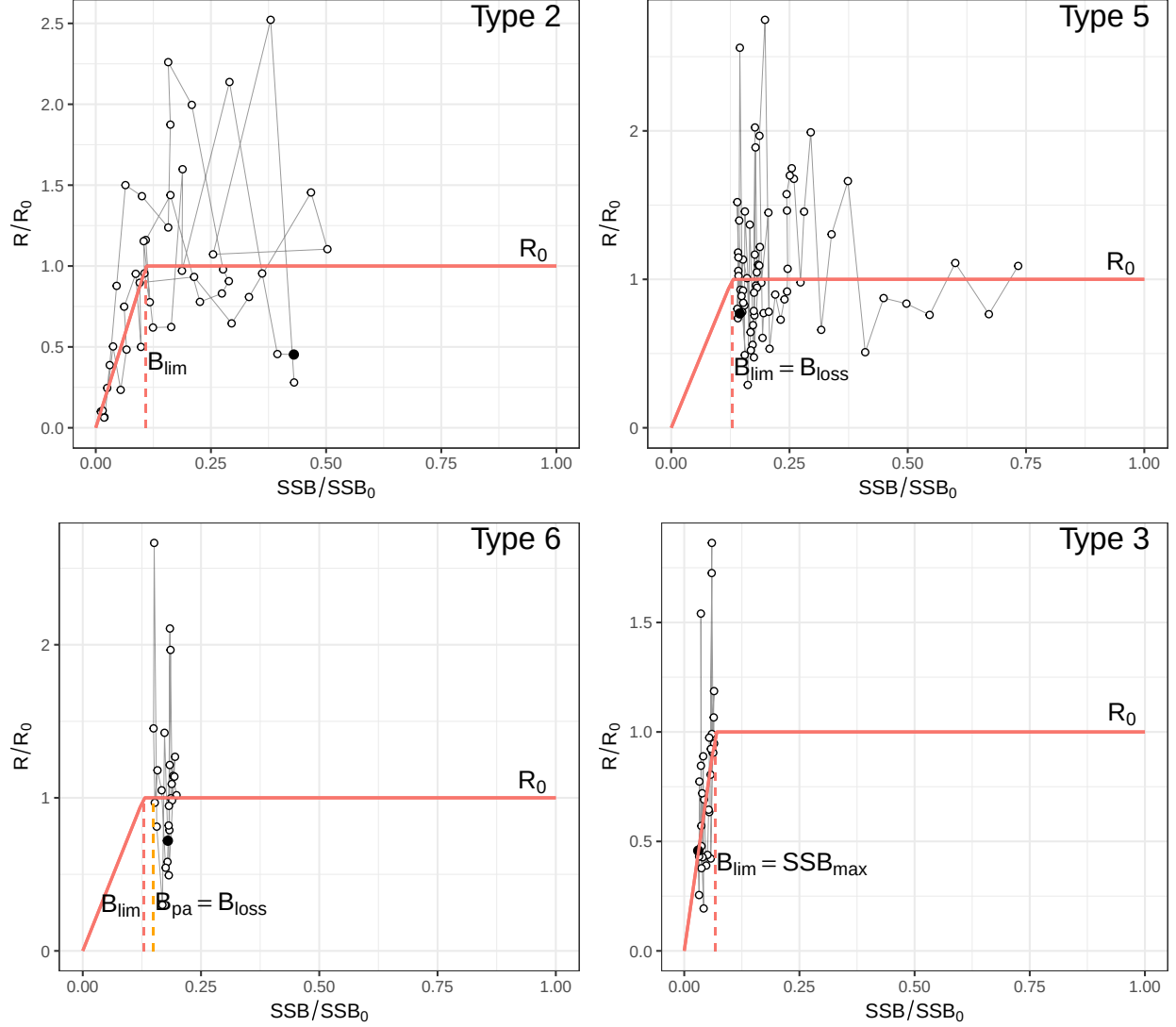


Figure 3: Illustration of four common S-R pattern types that are considered in ICES for deriving estimates of B_{lim} .

2.3.2 Beverton-Holt

For reference point estimation with FLBRP in FLR the Beverton and Holt SRR is formulated as:

$$R_y = \frac{aSB_{y-a_{min}}}{b + SBB_{y-a_{min}}}$$

where R_y is the number of recruits in year y , SSB_{y-a_r} is the spawning biomass in year y minus minimum age a_{min} defined for the stock (typically age-0 or age-1).

However, to provide flexibility for exploring alternative assumption about the steepness parameter, the Beverton-Holt equation in FLSRTMB is re-parameterised as function of steepness s and annual unfished spawning biomass per-recruit SPR_{0_y} (Mace and Doonan, 1988), such as routinely used in, e.g., Stock Synthesis (Methot and Wetzel, 2013):

$$R_y = \frac{4sSB_{y-a_{min}}R_0}{R_0SPR_{0_y}(1-s) + SBB_{y-a_{min}}(5s-1)}$$

where steepness s is defined as the ratio of recruitment when SSB equals 20% of the unfished SSB_0 to the virgin recruitment R_0 at SSB_0 and SPR_{0_y} is treated as non-stationary being function of annual quantities of $W_{a,y}$, $Mat_{a,y}$ and $M_{a,y}$. This formulation enables, for example, integration of available prior information on the steepness s of the SSR from a recent meta-analysis (Thorson 2020) or fix s for range of values to be explored (e.g. for sensitivity analysis of reference point estimation or operating model conditioning).

To account for the property that s is bounded by definition of the Beverton Holt between 0.2 and 1, The prior distribution for s is generated from truncated logit distributions (*TrunkLogit*).

$$s = 0.2001 + 0.7999/(1 + \exp(-s_{logit}))$$

such that

$$s \sim TrunkLogit(s_{logit}, \sigma_{logit})$$

where s_{logit} and σ_{logit} correspond to the input of species-specific predictions for the distribution of s from the hierarchical taxonomic FishLife model (Thorson, 2020).

The FLSRTMB estimates of R_0 and s are then converted into the parameters a and b of the Beverton-Holt formulation in FLR, such that

$$a = \frac{4sR_0SPR_0}{5sSPR_0 - 1}$$

\$

$$b = \frac{R_0SPR_0(1-s)}{5s-1}$$

where the reference for SPR_0 to predict a and b can be taken, e.g., as the average SPR_{0_y} over the last three years of the assessment model.

For an unconstrained estimation of s an approximately uniform prior distribution over a wide range of s is assumed by setting σ_{logit} to 20.

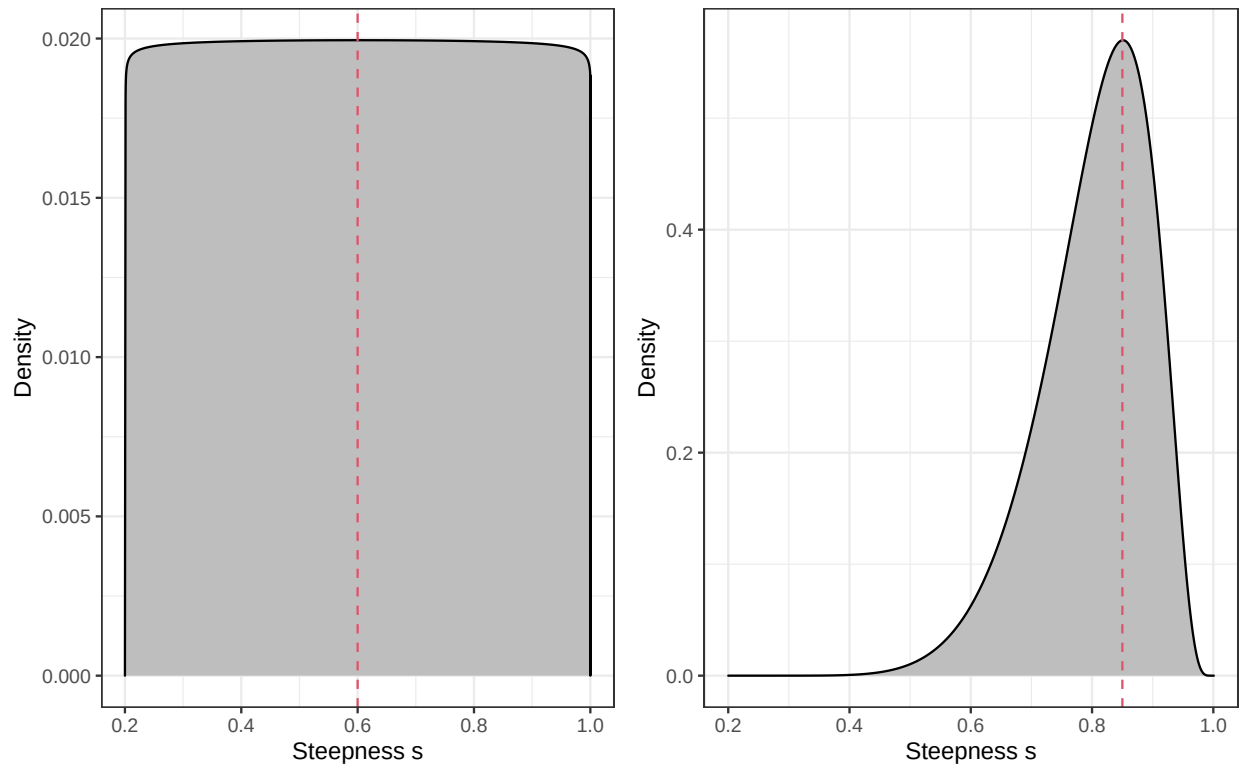


Figure 4: Illustration of the default setting in FLSRTMB for a mostly uninformative steepness s with a mean $s = 0.6$ with a standard deviation of 20 (left) and an example of an informative s prior with a mean $s = 0.85$ and standard deviation of 0.7 (right)

2.3.3 Ricker

Like the Beverton-Holt SRR, the Ricker SRR is formulated as function of the parameter a and b in FLR when used reference point estimation in $FLBRP$, such that:

$$R_y = aSB_{y-a_{min}}e^{-bSSB_{y-a_{min}}}$$

For consistency of parameterisation within $FLSRTMB$, the Ricker function parameters a and b are derived as a function of the estimable parameters s , R_0 and SPR_0 , given by:

$$b = \log(5.0 * s) / (0.8R_0SPR_0);$$

$$a = \frac{e^{bR_0SPR_0}}{SPR_0}$$

This formulation also enables fixing s or using informative priors for s for the Ricker SRR. Like the Beverton-Holt SRR s is bounded by definition at 0.2, but the Ricker model the upper bound can be set to very large value (here 20) to permit various degrees density dependence as SSB approaches SSB_0 .

2.4 Reference point estimation in FLRef

For the estimation and evaluation reference points, the R package **FLRef** (Winker) was developed and is available on <https://github.com/Henning-Winker/FLRef>. This package is implemented with **FLR** and makes of the optimisation routine for estimating fisheries reference points at equilibrium that is available in **FLBRP**. **FLRef** requires the stock data as **FLStock** objects and the stock recruitment functions in the form of **FLSR** objects, which were produced with the package **FLSRTMB** (Winker and Mosqueira). The EWG Expert Henning Winker (JRC) developed tested the package on the stock data prior to the meeting with the support of the FLR Core Team Members Iago Mosqueira (Wageningen University) and Laurence Kell (Imperial College).

Main features of **FLRef** include:

- Automation of estimating and visualizing a wide range of limit and target plots
- New plotting options for illustrating reference points both as relative or absolute quantities
- New plotting options for compare the impact of various stock recruit model assumptions on the estimated reference points
- Newly designed fisheries advice plots

3 Red Mullet

Subset red mullet stocks

```
stocks = stks[8:12]

# Select stock

stk = stocks[[1]]
stk@name = gsub("\\_", ".", stk@name)
```

3.1 MUT01

3.1.1 Stock dynamics of MUT01

Notable features of the MUT01 stock are:

- The stock trajectory are highly smoothed and follow a cyclic pattern
- Weight-at-age shows some variation, in particular in the plus group
- High M_a in particular for ages 1-2
- Biomass largely driven by ages 1-2.
- MUT01 is only fully selected at age-3
- Late selectivity implies that the large proportion of biomass (ages1-2) is not vulnerable to fishing
- It also implies that most fish can mature before being caught so that risk of growth overfishing would be unlikely

```
plot(stk) + theme_bw() + facet_wrap(~qname, scales = "free", ncol = 2) + ggtitle(paste0(stk@name,
  " (", stk@species, ")"))
```

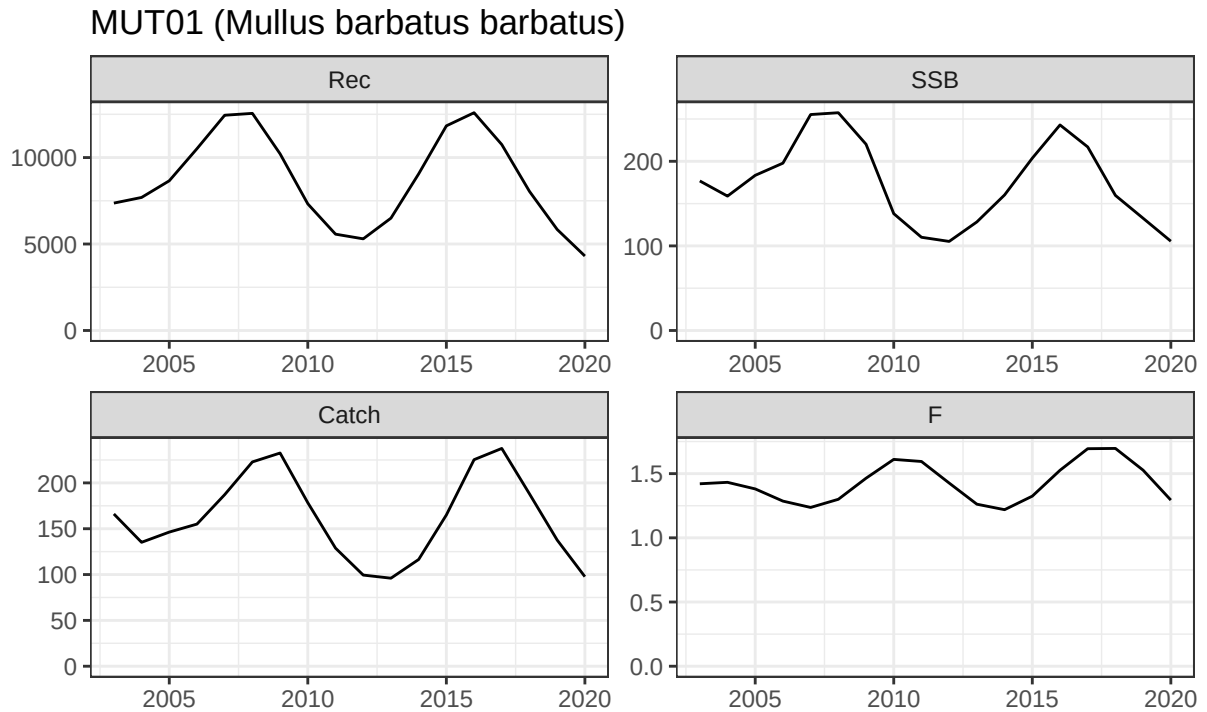


Figure 5: Estimated stock assessment trajectories for MUT01

```
plotdyn(stk) + ggtitle(paste0(stk@name, " (", stk@species, ")"))
```

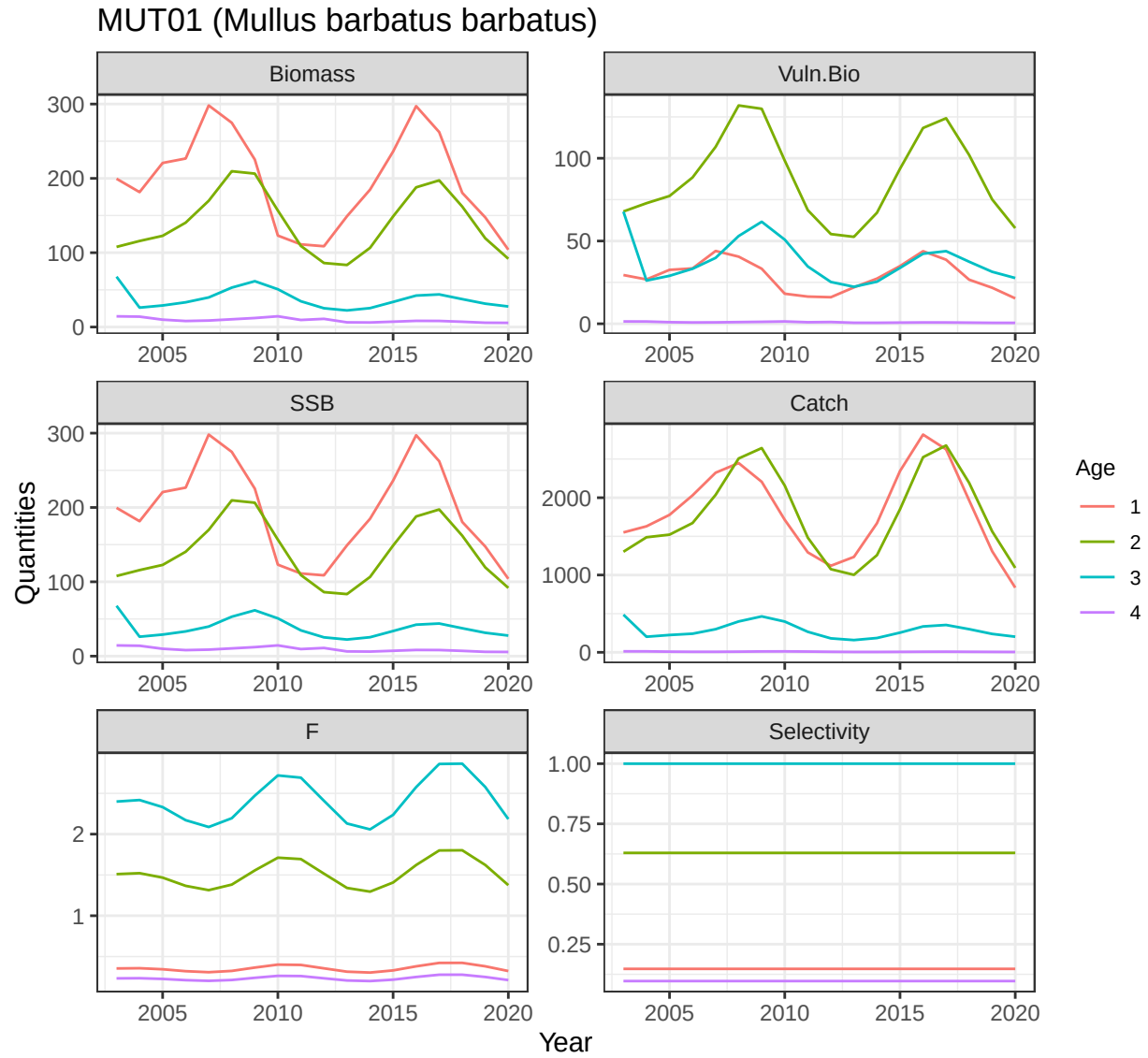


Figure 6: Stock assessment trajectories at age for MUT01

```
plotbiogr(stk) + ggtitle(paste0(stk@name, " (", stk@species, ")"))
```

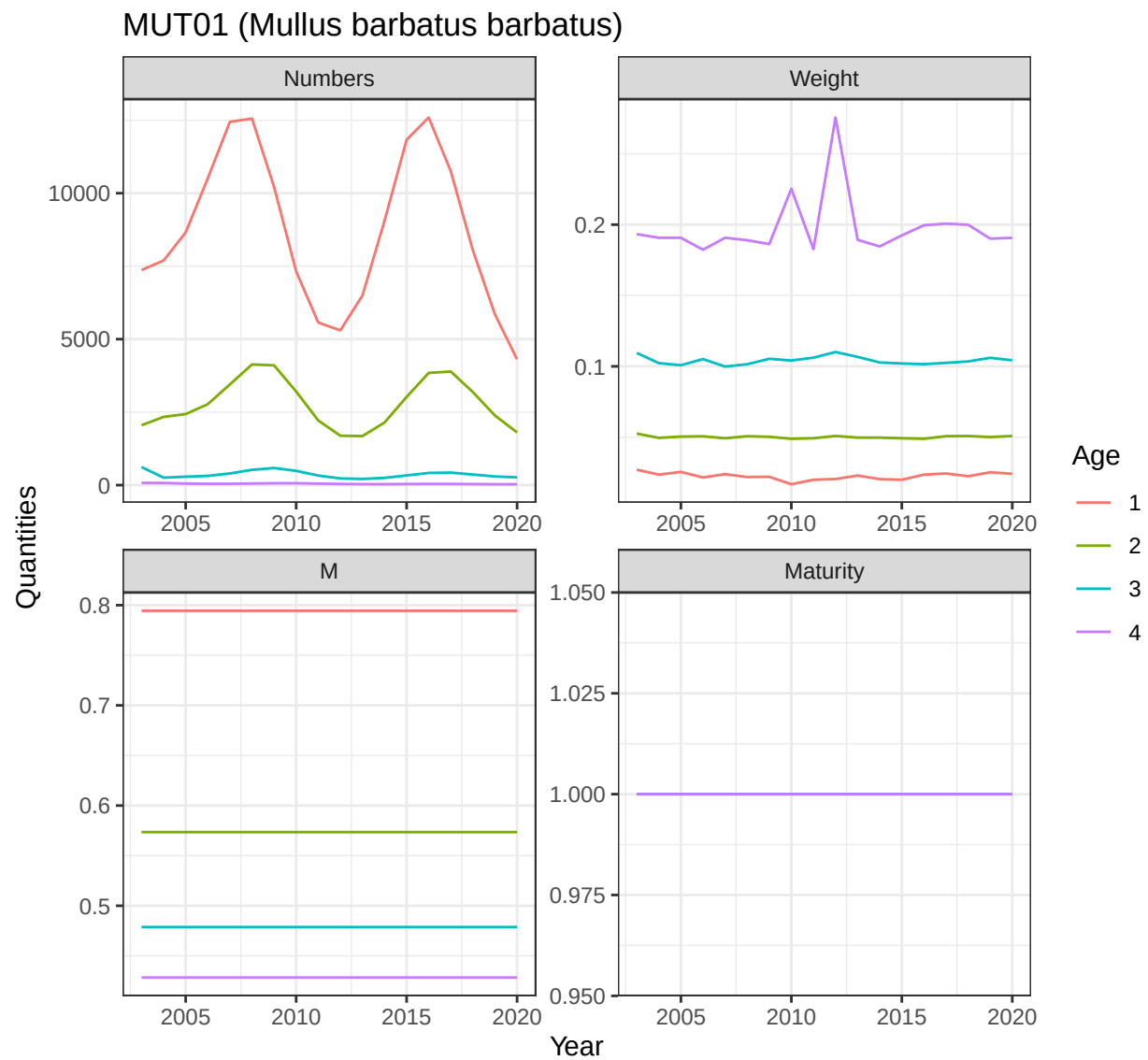


Figure 7: Stock biology trajectories at age for MUT01

```
plotbioage(stk) + ggtitle(paste0(stk$name, " (", stk@species, ")"))
```

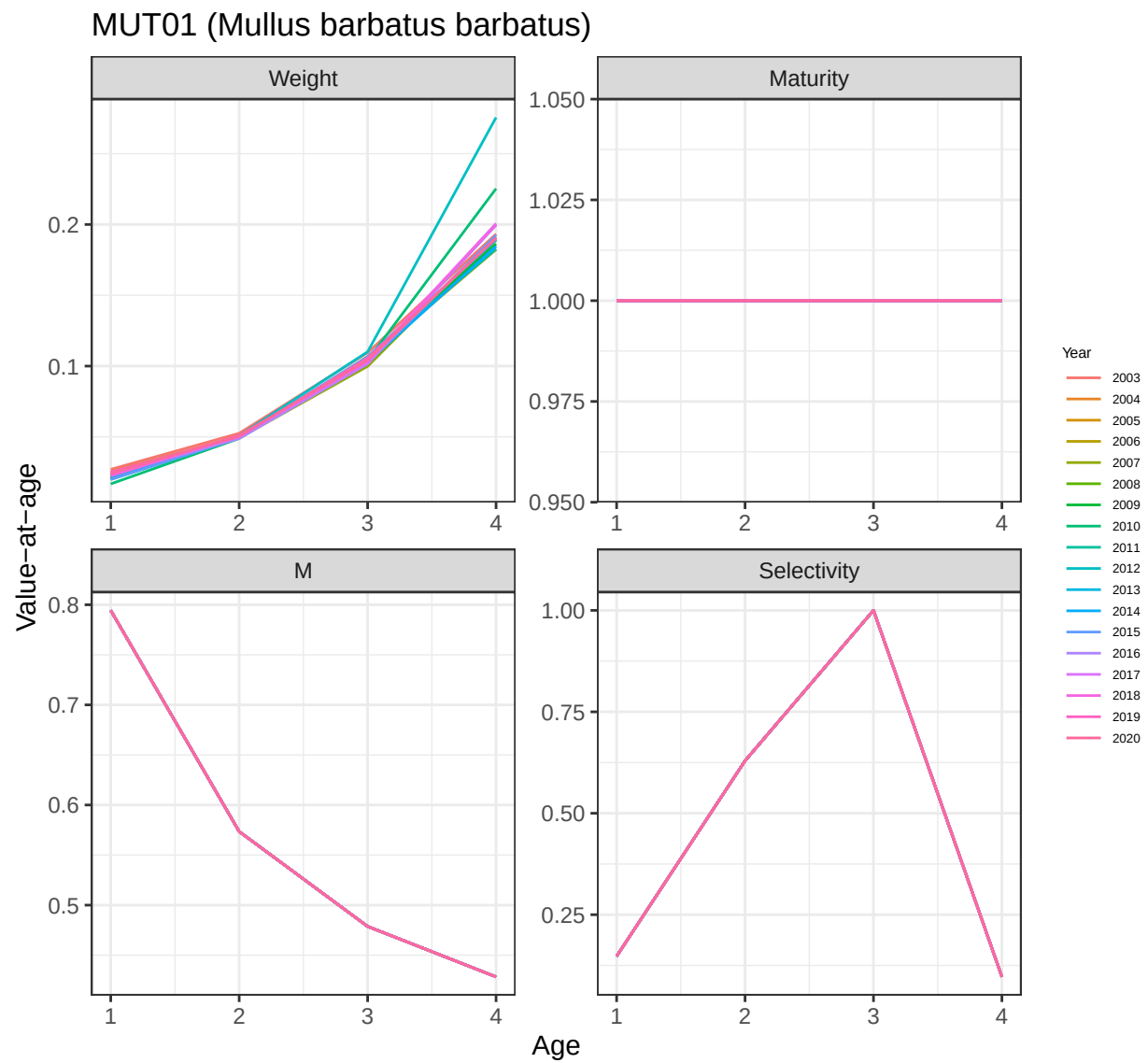


Figure 8: Annual stock quantities at age for MUT01

3.1.2 Per-recruit Analysis for MUT01

Compute the per-recruit reference points of interest

```
pr = computeFbrp(stk, proxy = "f0.1", blim = 0.25, type = "btrg", verbose = T)
Computing Per-Recruit Quantities
Computing F0.1 with Btrg = B[F.01]

Blim = 0.25 with Btrg corresponding to F0.1

# check
Fbrp(pr)
An object of class "FLPar"
params
  F0.1   Btrg   Blim   Flim   Yeq   B0
0.6074 0.0405 0.0127 3.0357 0.0168 0.1250
units:  NA
```

The following observations are noted from the below results of the per-recruit analysis for MUT01:

- The asymptotic yield curve suggests that the stock cannot be growth over-fished, mostly as a result of high M_a and $Mat_a < Sel_a$.
- Maximum yield per recruit (YPR) is attained as F approach infinity
- As a result YPR is higher at the B_{lim} proxy of $0.25B_{F0.1}$ (approx. $0.1SPR_0$) than at $F_{0.1}$
- The age-structure is strongly truncated compared to unfished levels at $F = 0$, corresponding to spawning potential ratio less than $SRP = 0.2$

```
ploteq(pr) + ggtitle(paste0(stk@name, " (", stk@species, ")"))
```

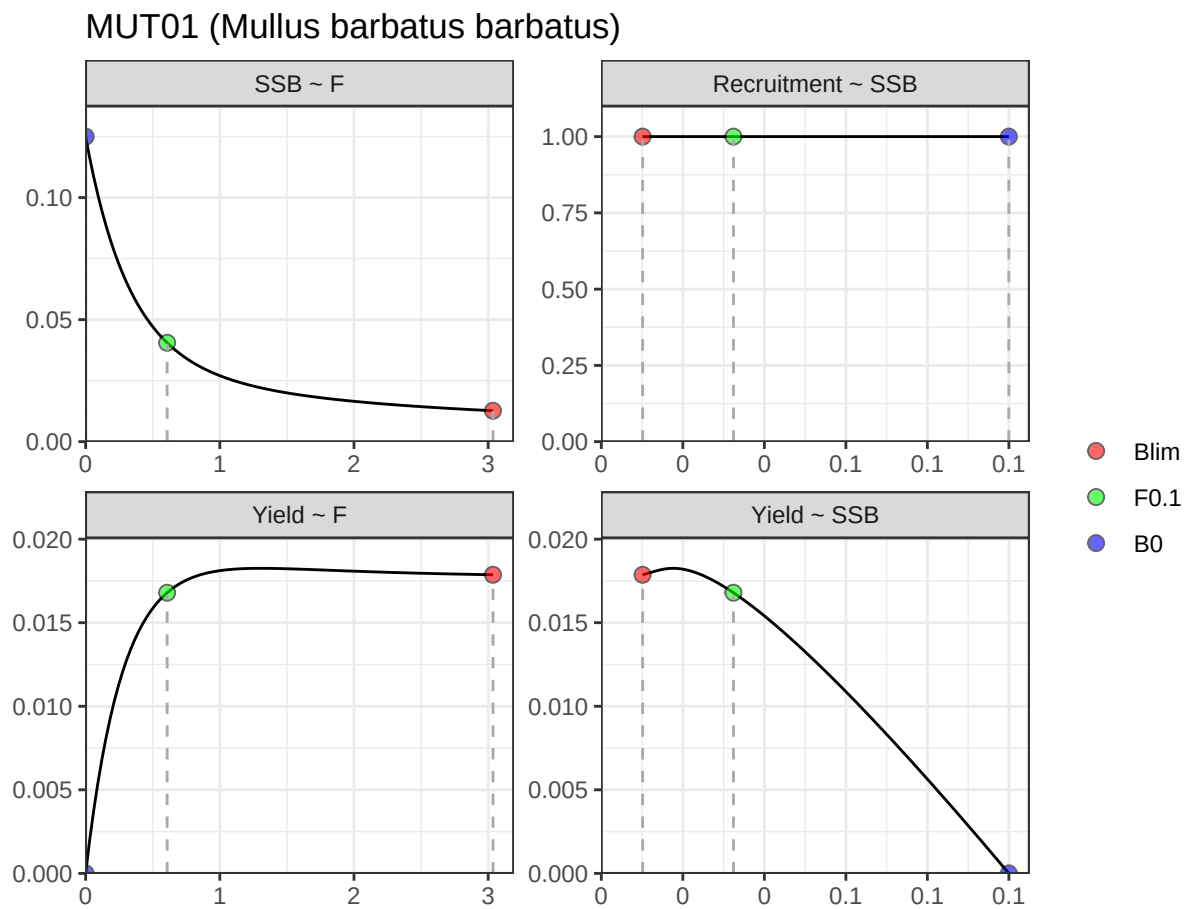


Figure 9: Per-recruit analysis for MUT01

```
plotspr(stk) + ggtitle(paste0(stk@name, " (", stk@species, ")"))
```

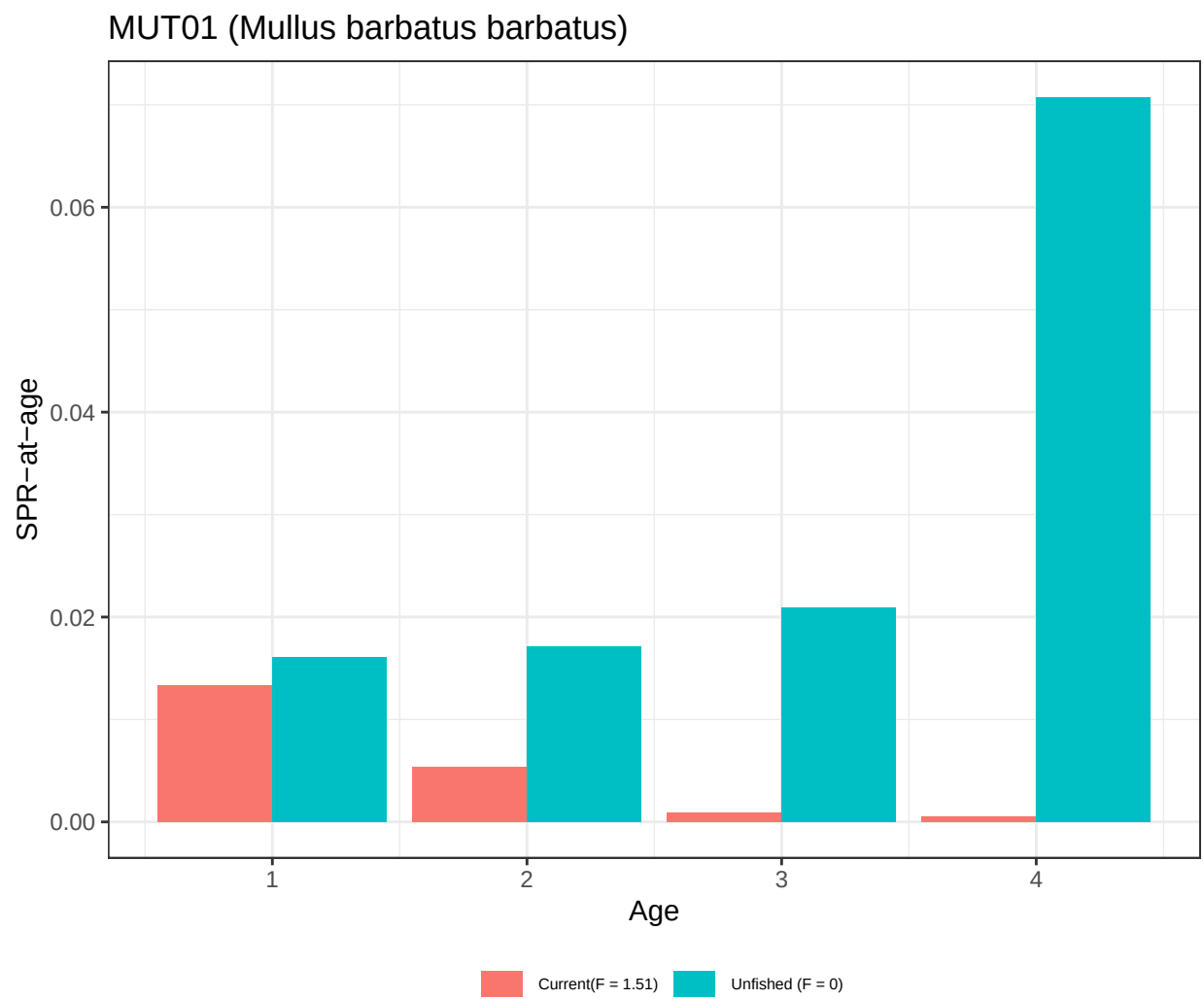


Figure 10: Comparison of the spawning biomass per recruit SPR_F at current F (average of last 3 years) and SPR_0 with $F = 0$ for MUT01

```
plotAdvice(stk, pr) + ggtitle(paste0(stk$name, " (", stk@species, ")"))
```

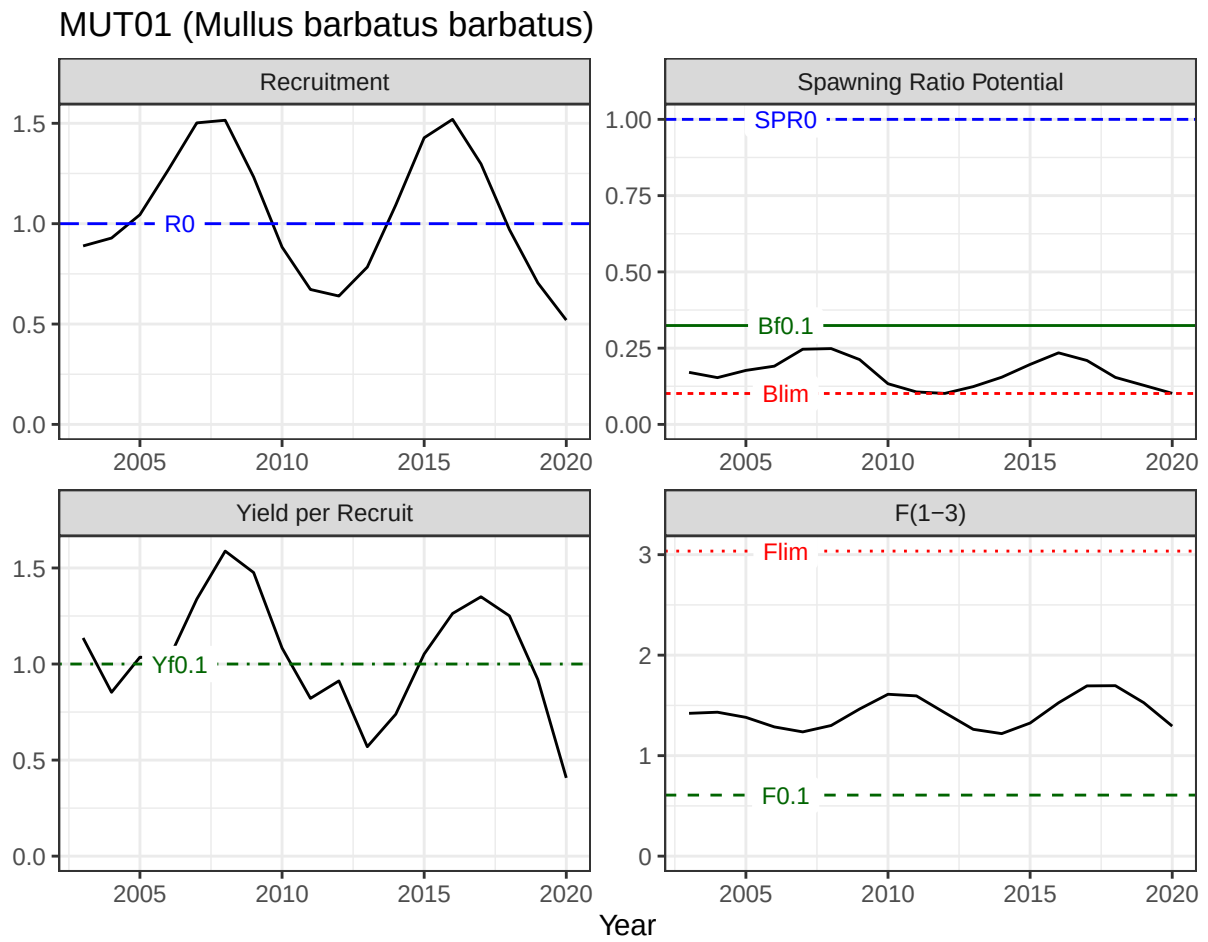


Figure 11: Time-series of stock quantities expressed as per-recruit for MUT01

3.1.3 Fitting stock-recruitment functions for MUT01

Initially four recruitment functions are explored, using the function `srrTBM` in the package `FLSRTMB`:

1. Geometric Mean (`model=geomean`)
2. Hockey-Stick (`model=segreg`)
3. Beverton-Holt (`model=bevholtSV`)
4. Ricker (`model=ricker`)

The Hockey-Stick is constrained to search for solutions of the break-point so that it falls within a lower bound (`lplim`) and upper bound (`uplim`) of spawning ration potential $SRP_{lim} = SPR_{lim}/SPR_0$. Note that in the specific case of the Hockey-Stick $SRP_{lim} = B_{lim}/B_0$, but SRP_{lim} can be generalised even if the $b = B_{lim}$ estimate is inputted into another S-R. The initial bounds are chosen to be fairly on constrained for a range of $SRP_{lim} = SPR_{0.1-20}$ by setting `lplim=0.001` and `uplim=0.2`.

In the initial fits of the Beverton-Holt and Ricker models steepness s and $R0$ are estimated given the input SPR_{0_y} .

```
# compute annual spr0y
spr0 = spr0y(stk)
# Geometric Mean
gm = srrTMB(as.FLSR(stk, model = geomean), spr0 = spr0, nyears = 3)
# Constraint Hockey-Stick
hs = srrTMB(as.FLSR(stk, model = segreg), spr0 = spr0, lplim = 0.001, uplim = 0.2,
  nyears = 3)
# Beverton Holt
bh = srrTMB(as.FLSR(stk, model = bevholtSV), spr0 = spr0, nyears = 3)
# Ricker
ri = srrTMB(as.FLSR(stk, model = rickerSV), spr0 = spr0, nyears = 3)

# Estimates
gm@SV
  s      sigmaR      R0      rho      B0
1 NA 0.3360107 8343.905 0.7464474 1052.396
hs@SV
  s      sigmaR      R0      rho      B0
1 NA 0.2472207 10350 0.6582059 1305.419
bh@SV
  s      sigmaR      R0      rho      B0
1 0.6806813 0.24641 25998.19 0.708014 3279.086
ri@SV
  s      sigmaR      R0      rho      B0
1 0.9877606 0.2479318 15138.65 0.7440852 1909.4

# Combine in one object
srs = FLSRs(gm = gm, hs = hs, bh = bh, ri = ri)
```

```

p1 = plotsrs(srs)
p2 = plotsrts(srs)
p3 = plotsrs(srs, b0 = T)
p4 = plotsrs(srs, b0 = T, rel = T)

```

```

ggarrange(p1, p2, p3, p4, ncol = 2, nrow = 2, common.legend = TRUE, legend = "right")

```

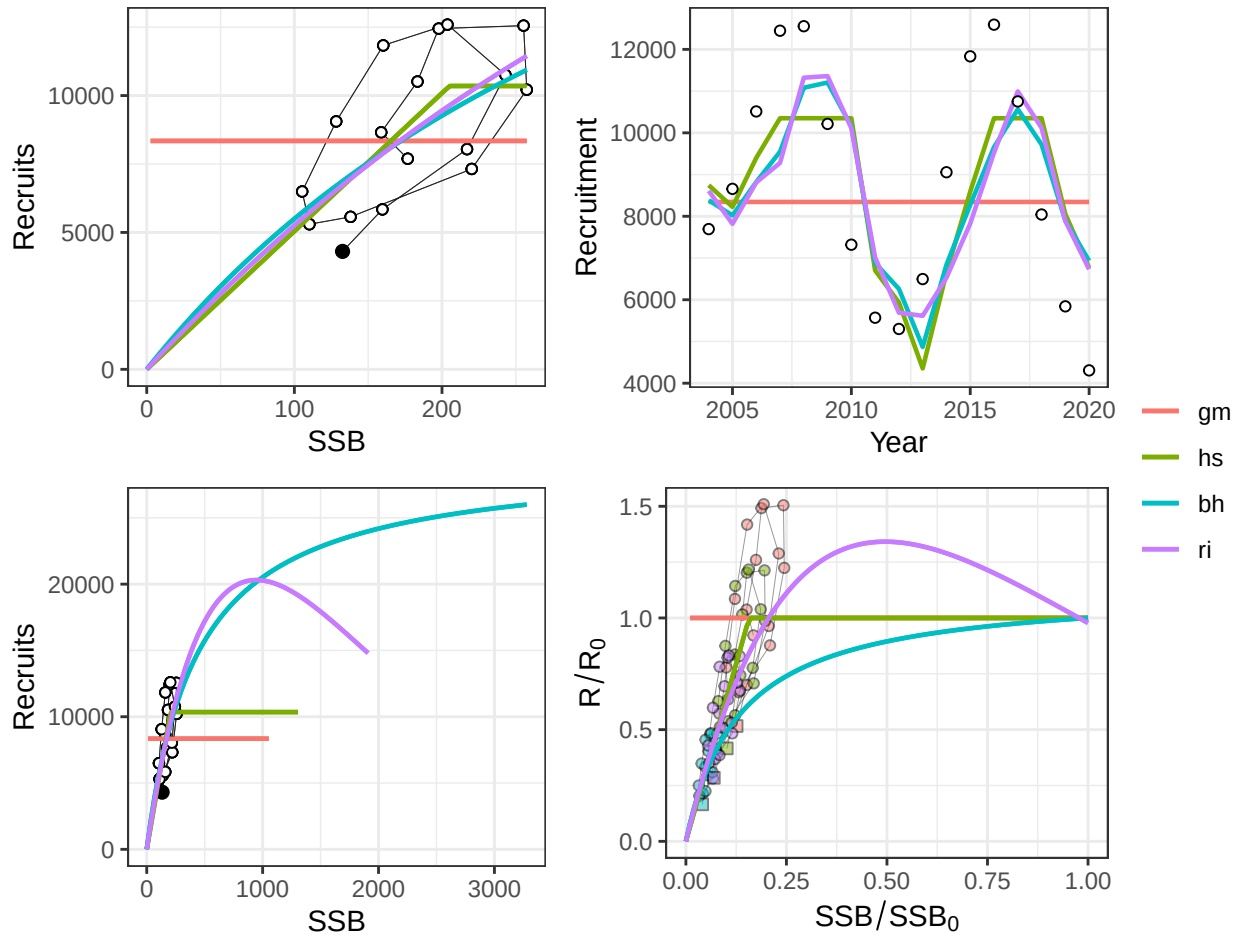


Figure 12: Fits of alternative stock-recruitment relationship for MUT01 assuming a geometric mean (gm), conditioned hockey-stick (hs), beverton-holt (bh) and ricker (ri) model

The results show that the recruitment variation fairly low, e.g. $\sigma_r = 0.25$ for the Beverton-Holt model, associated with a steepness of $s = 0.68$. The predicted recruitment by Hockey-Stick, Beverton-Holt and Ricker models follow the observed recruitment pattern over time, however, the three model differ largely in scale of their R_0 and B_0 estimates.

The break-point of the hockey-stick is estimated at $b = 205$ corresponding to an $SRP_{lim} = 0.157$ and the corresponding R_0 is slightly above the geometric mean recruitment.

In the following the sensitivity to alternative fixed steepness values of $s = 0.8 - 0.95$ for the Beverton-Holt model explored. The results show that increasing $s = 0.68$ to $0.8-0.85$ substantially decrease the R_0 and B_0 estimates to a scale that is comparable to the Hockey-Stick estimates.

```
# range of s
s = seq(0.8, 0.95, 0.05)
bhs = FLSRs(lapply(s, function(x) {
  srrTMB(as.FLSR(stk, model = bevholtSV), spr0 = spr0y(stk), s.est = FALSE, s = x)
}))

# require unique names
names(bhs) = paste0("s=", s)

# Add Geomean and Hockey-Stick SRR
srs[1:2]@names
[1] "gm" "hs"

bhs = FLSRs(c(srs[1:2], bhs))

p1 = plotsrs(bhs, b0 = F)
p2 = plotsrts(bhs)
p3 = plotsrs(bhs, b0 = T)
p4 = plotsrs(bhs, b0 = T, rel = T)
ggarrange(p1, p2, p3, p4, ncol = 2, nrow = 2, common.legend = TRUE, legend = "right")
```

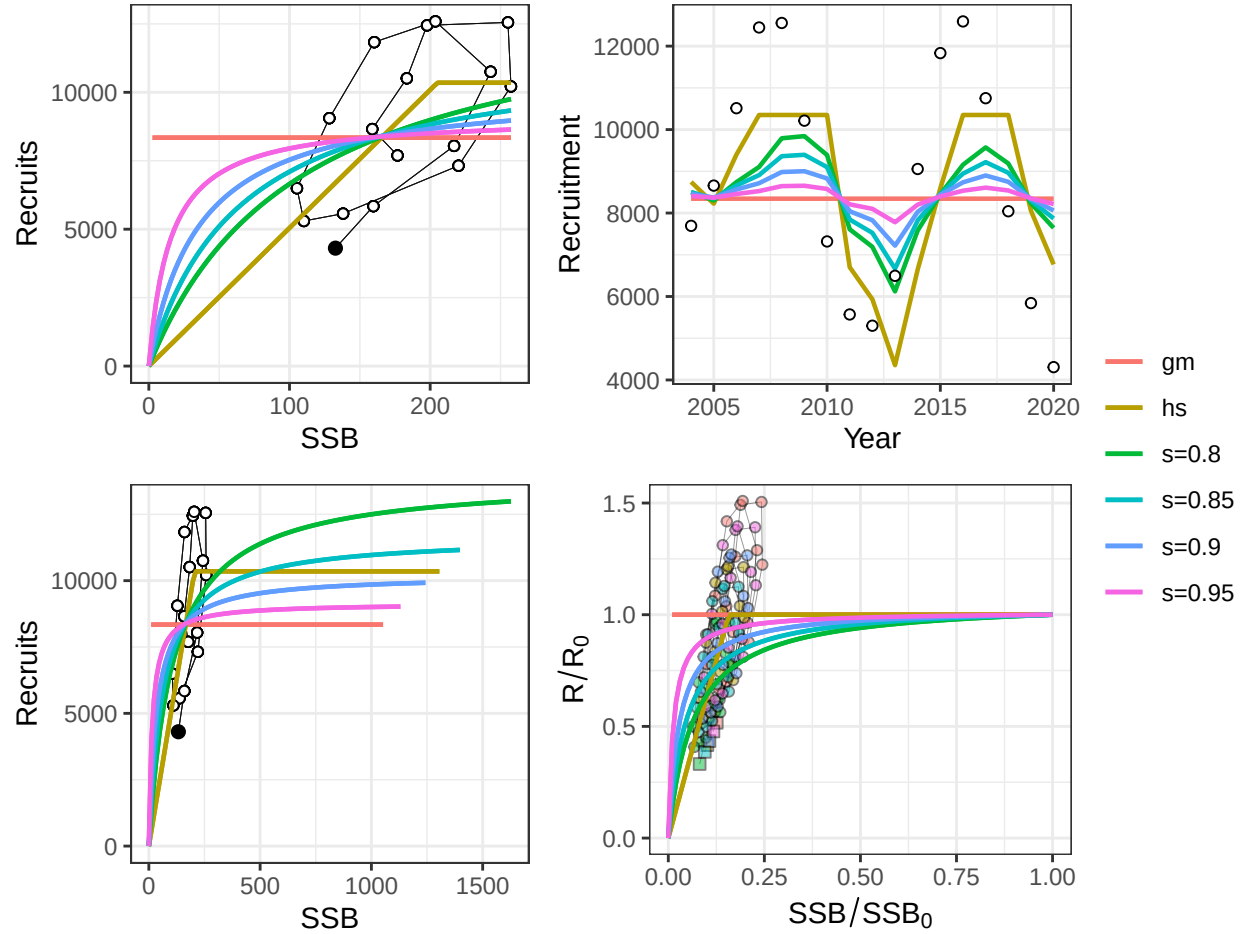


Figure 13: Fits of stock-recruitment relationships for MUT01 assuming a geometric mean (gm), conditioned hockey-stick (hs), alternative steepness s values for beverton-holt (bh) model

3.1.4 Reference point estimation for MUT01

The following candidate S-R models are considered for initial reference point estimation:

1. Hockey-Stick with B_{lim} set to the break-point $b = 205$
2. Beverton-Holt with a fixed $s = 0.85$ with $B_{lim} = 0.25B_{F0.1}$ ⁴

```
# Select S-R models
selsr = FLSRs(c(srs[2], bhs[4]))
# Extract blim
blim = an(hsblim(hs)["Blim"])
# set up varying Blim parameterisation
blims = c(blim, 0.25)
types = c("value", "btrg")

brps = FLBRPs(Map(function(x, y, z) {
  computeFbrp(stk, x, proxy = "f0.1", blim = y, type = z, verbose = FALSE, fmax = 10)
}, x = selsr, y = blims, z = types))
```

```
ploteq(brps, obs = T)
```

```
ploteq(brps, obs = T, rel = T)
```

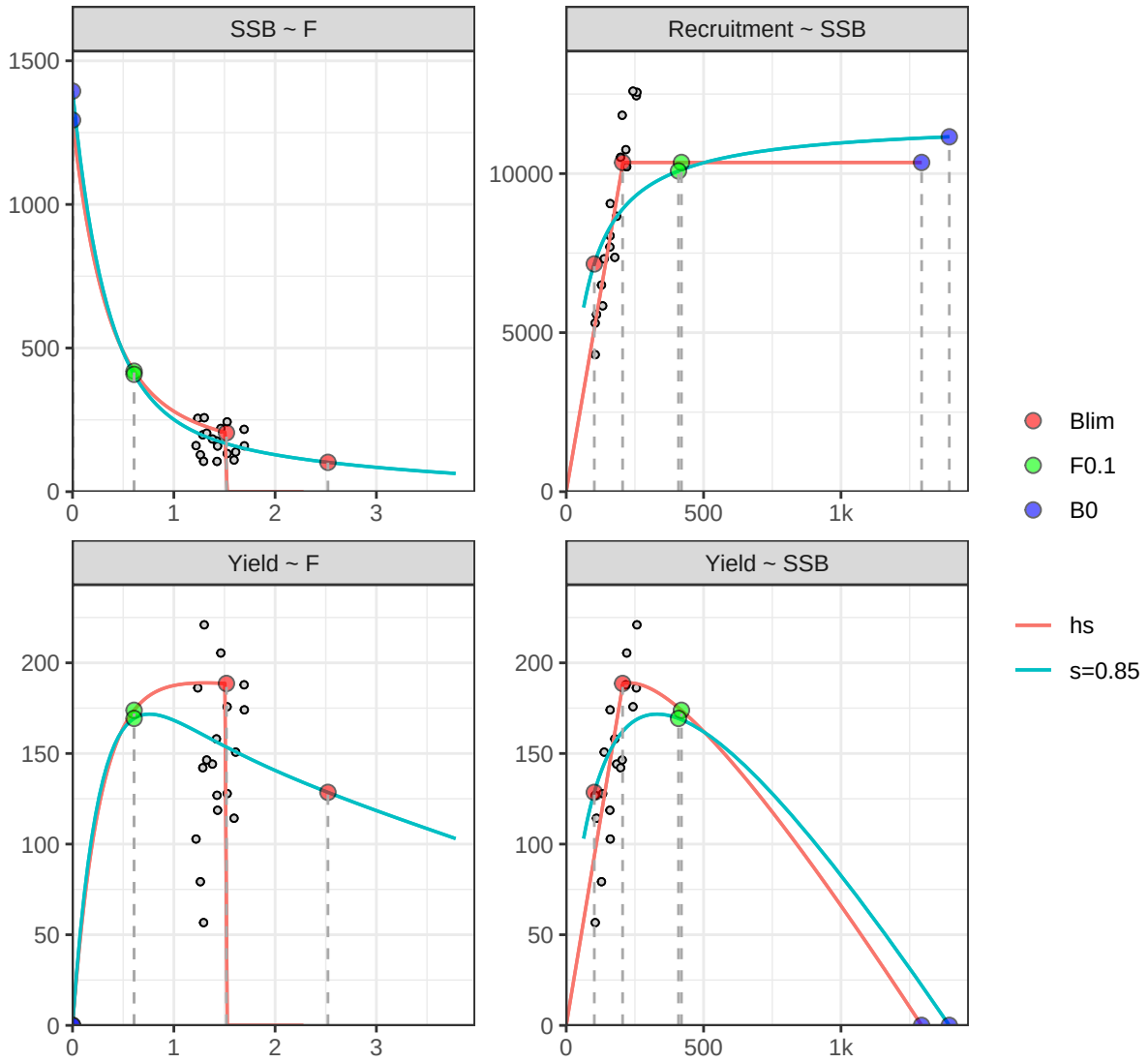


Figure 14: Reference point estimates of $F_{0.1}$, B_{lim} and B_0 shown as functions of SSB , F , Yield and Recruitment. Grey dots show the corresponding observations for MUT01

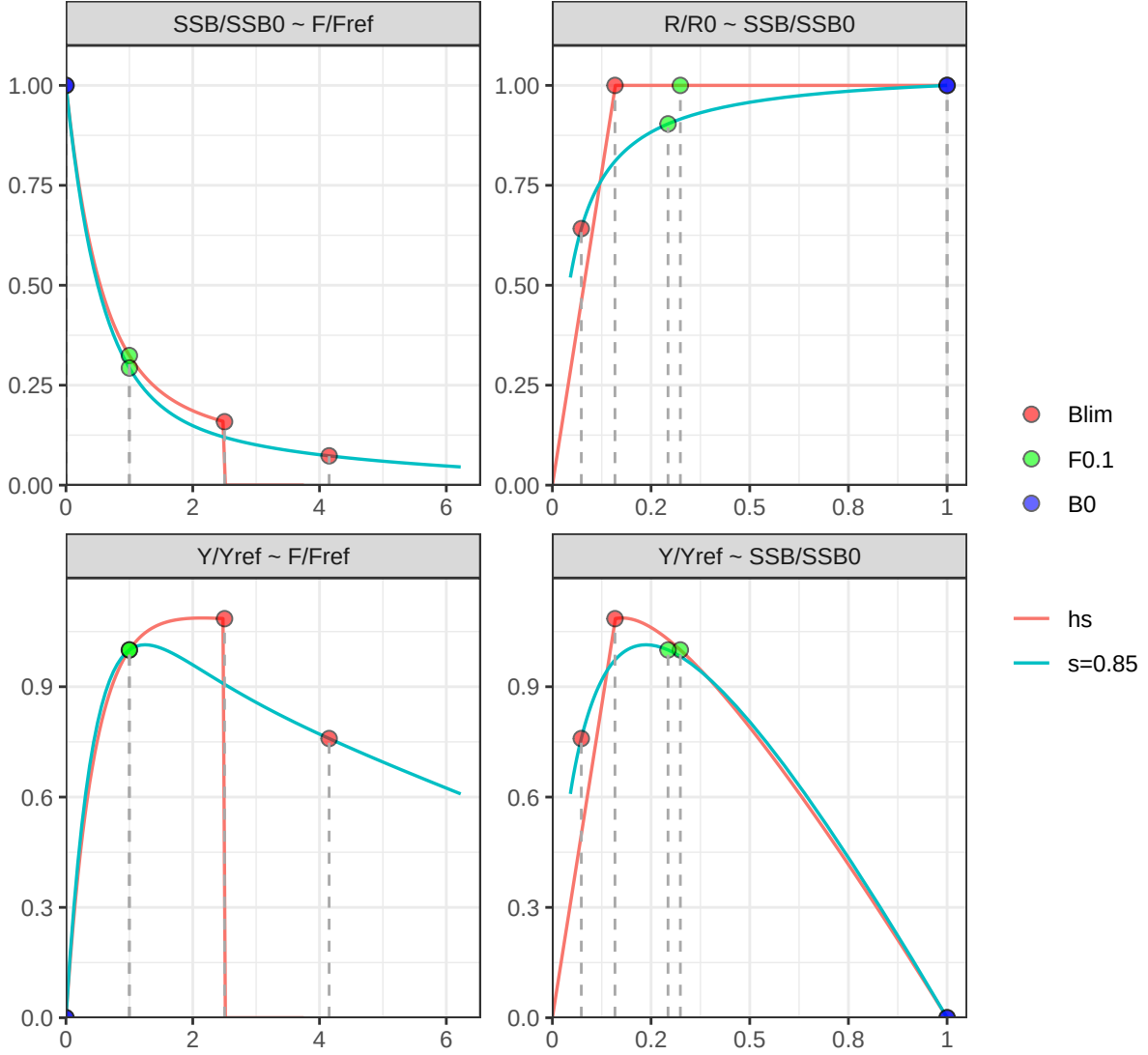


Figure 15: Relative reference point estimates of F_{01} , B_{lim} and B_0 shown as functions of SSB/SSB_0 , $F/F_{0.1}$, $Yield/Y_{F_{0.1}}$ and R/R_0 . Grey dots show the relative values of the corresponding observations for MUT01

3.1.5 Implications for stock status of MUT01

```
plotAdvice(stk, brps$hs) + ggtitle(paste0(stk@name, ": Hockey-Stick"))
```

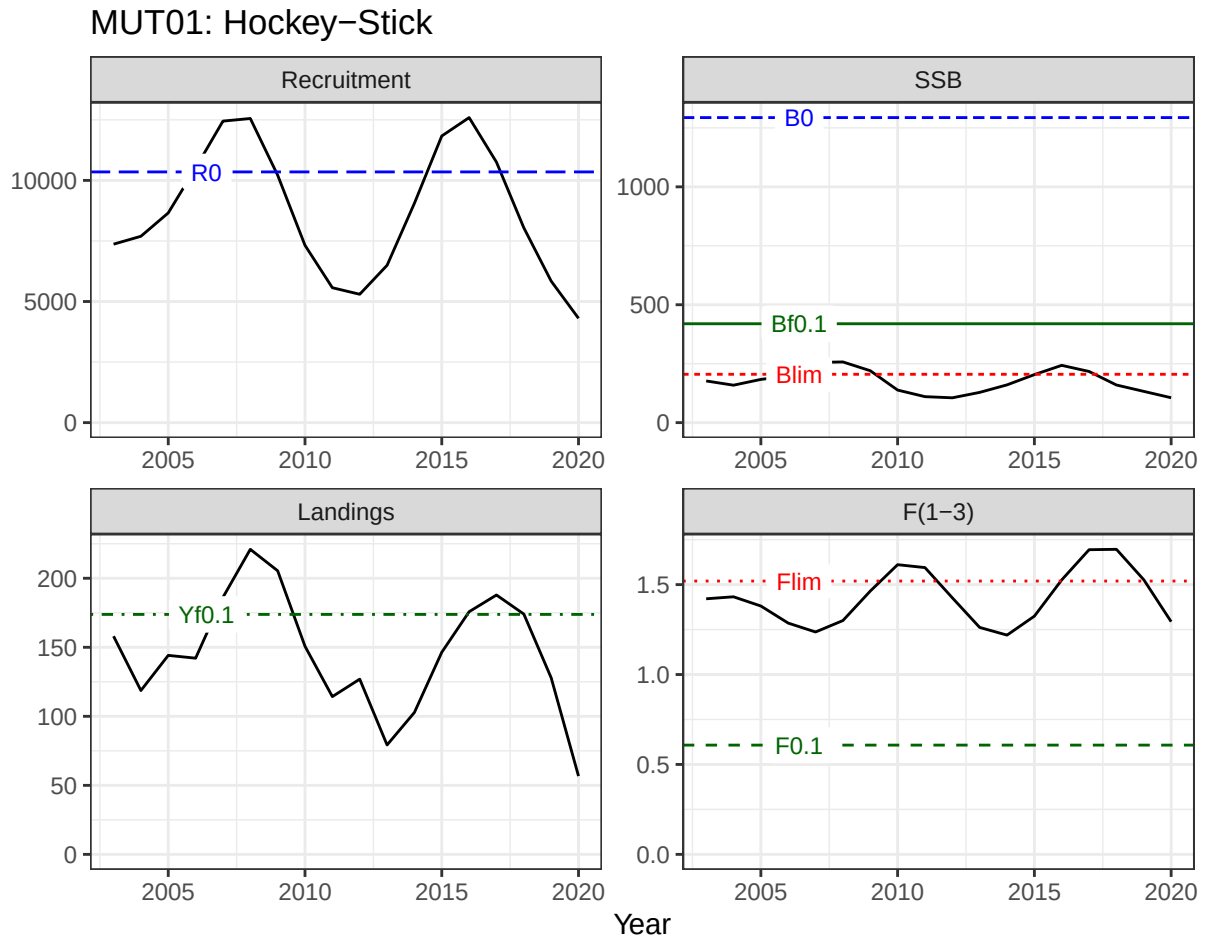


Figure 16: Status Advice plot for MUT01 showing stock trajectories of Recruitment, SSB , F , Landings and $Yield$ compared to the estimated reference points, based on a Hockey-Stick stock-recruitment relationship

```
plotAdvice(stk, brps[[2]]) + ggtitle(paste0(stk@name, ": Beverton-Holt"))
```

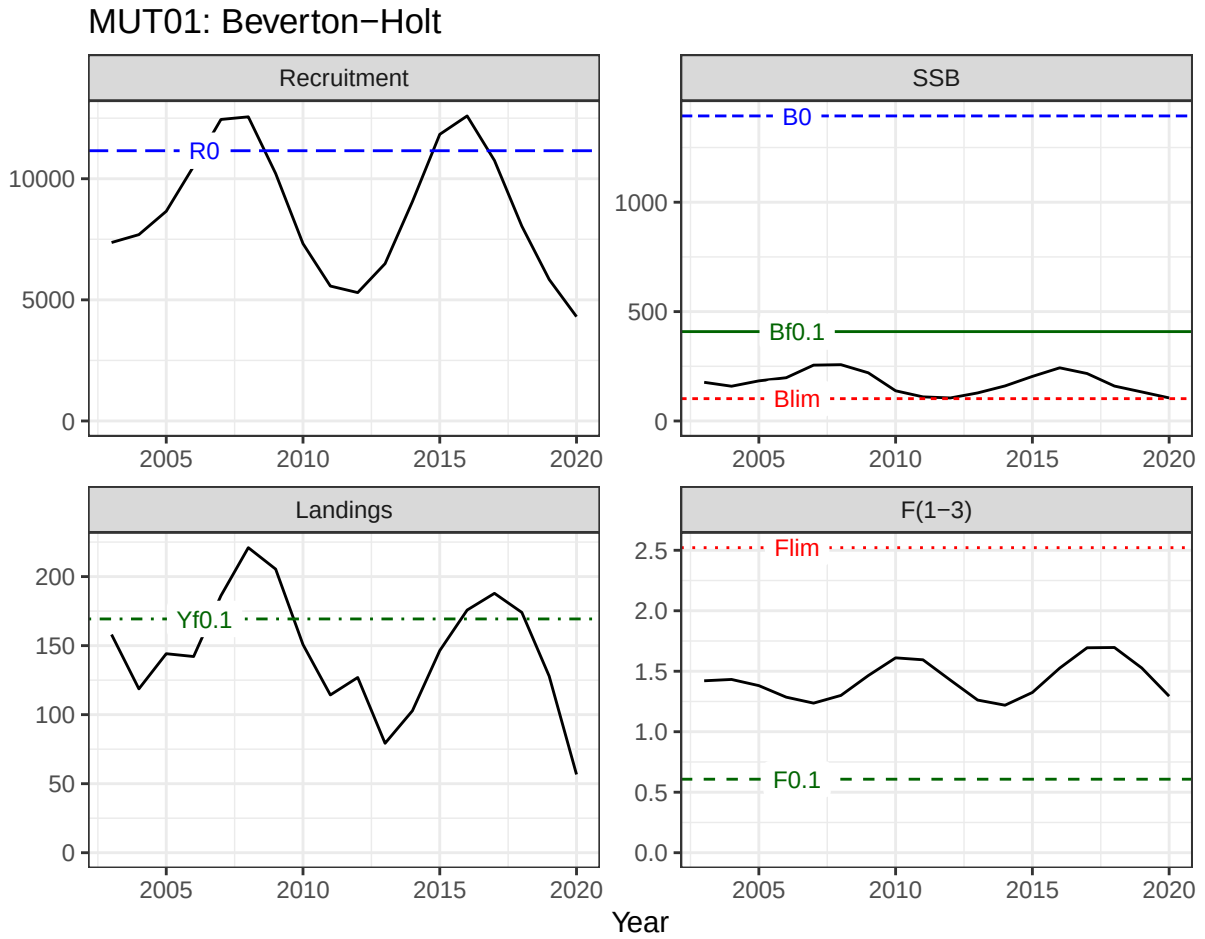


Figure 17: Status Advice plot for MUT01 showing stock trajectories of Recruitment, SSB , F , Landings and $Yield$ compared to the estimated reference points, based on a Beverton-Holt stock-recruitment relationship

```

p1 = plotAR(Fbrp(brps[[1]]), bpa = 1.4, kobe = FALSE, obs = stk) + ggtitle("a) Hockey-Stick (Absolute)")
p2 = plotAR(brps$hs, bpa = 1.4, rel = TRUE, kobe = FALSE, obs = stk) + ggtitle("b) Hockey-Stick (Relative)")
p3 = plotAR(brps[[2]], bpa = 1.4, kobe = FALSE, obs = stk) + ggtitle("c) Beverton-Holt (Absolute)")
p4 = plotAR(brps[[2]], bpa = 1.4, rel = TRUE, kobe = FALSE, obs = stk) + ggtitle("d) Beverton-Holt (Relative)")
ggarrange(p1, p2, p3, p4)

```

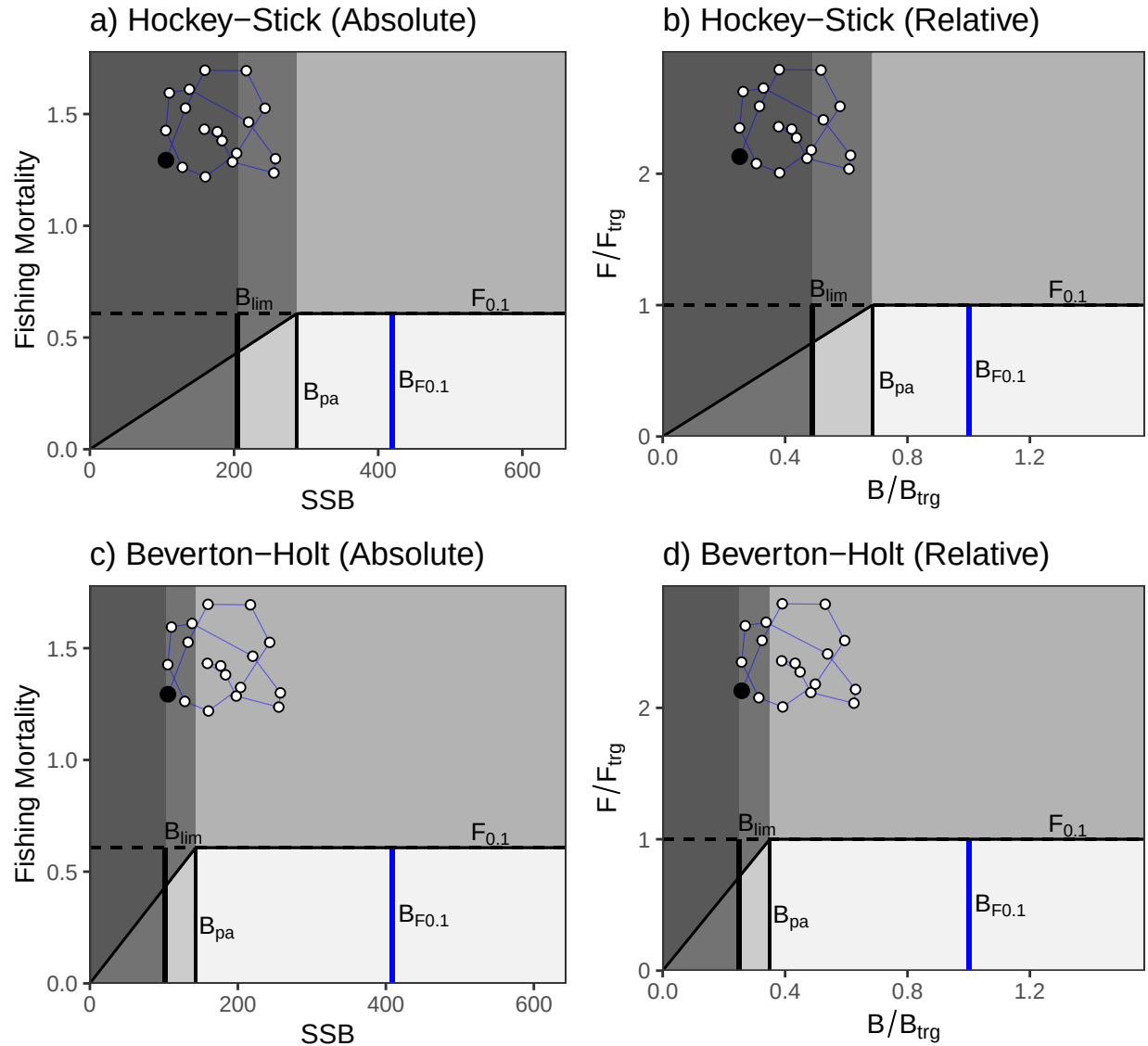


Figure 18: Advice Rule plots for MUT01, with $B_{pa} = 1.4B_{lim}$, showing a comparison between a Hockey-Stick and Beverton-Holt S-R with an empirical $B_{lim} = 0.25B_{F0.1}$.

3.2 MUT10

3.2.1 Stock dynamics of MUT10

Notable features of the MUT10 stock are:

- *SSB* shows a continuous increase while *F* decreases
- Very low weight at age of 90g for age-5
- Natural mortality is assumed notably higher than for other red mullet stocks
- MUT10 is fully selected at age-2, so a large proportion can mature (age-1) before becoming vulnerable to fishing.
- risk of growth over-fishing is unlikely

```
plot(stk) + theme_bw() + facet_wrap(~qname, scales = "free", ncol = 2) + ggtitle(paste0(stk@name,
  " (", stk@species, ")"))
```

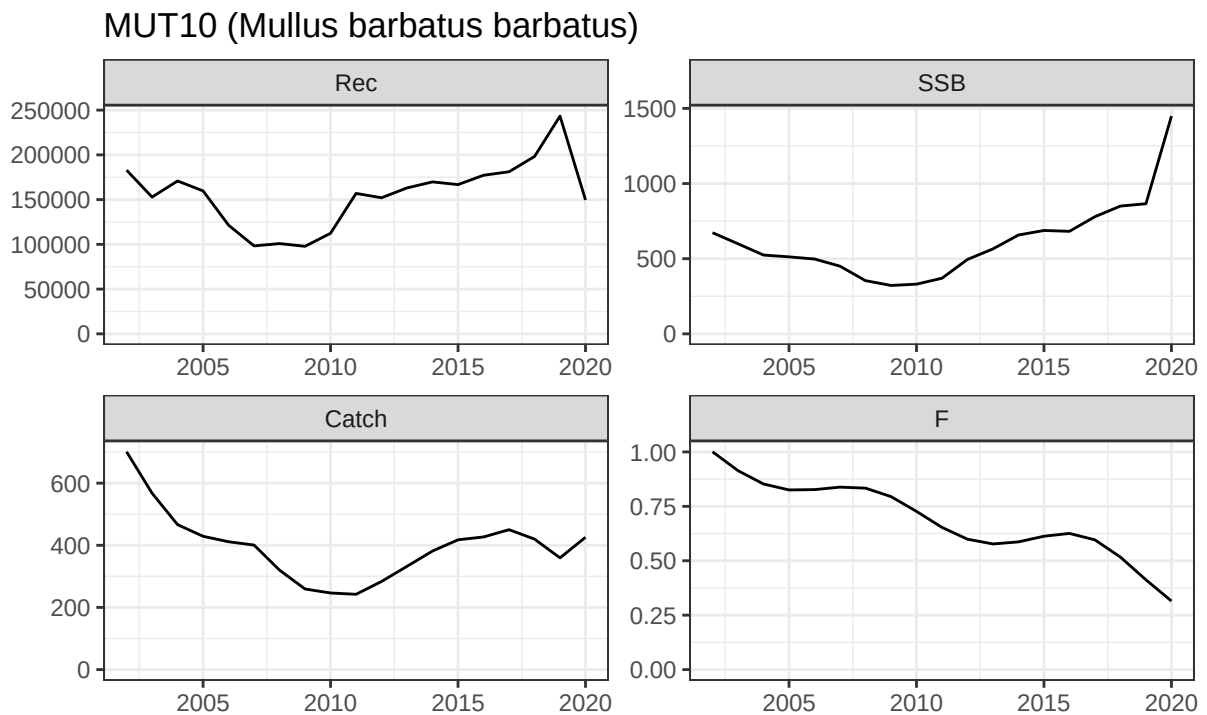


Figure 19: Estimated stock assessment trajectories for MUT10

```
plotdyn(stk) + ggtitle(paste0(stk@name, " (", stk@species, ")"))
```

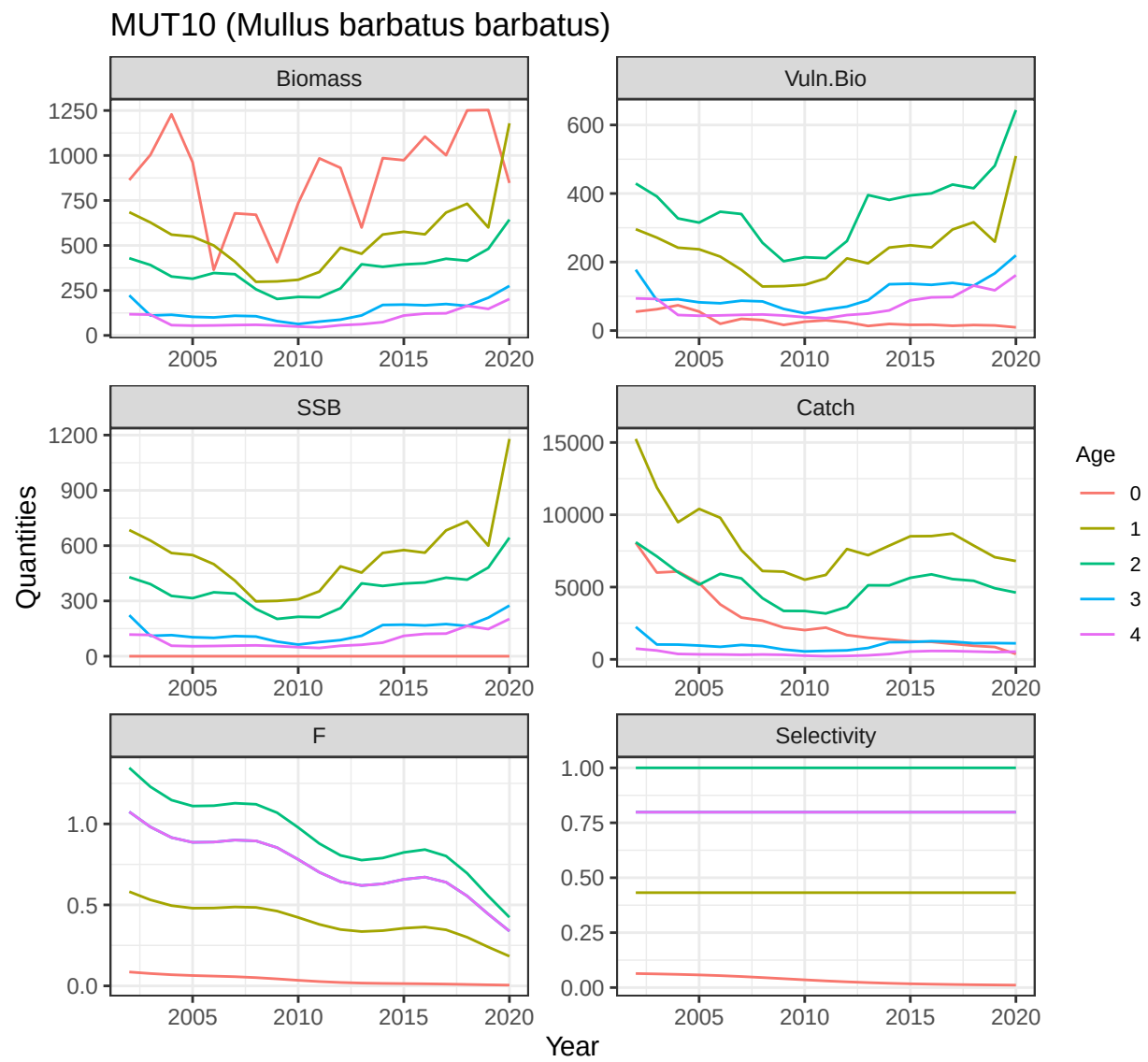


Figure 20: Stock assessment trajectories at age for MUT10

```
plotbiogr(stk) + ggtitle(paste0(stk@name, " (", stk@species, ")"))
```

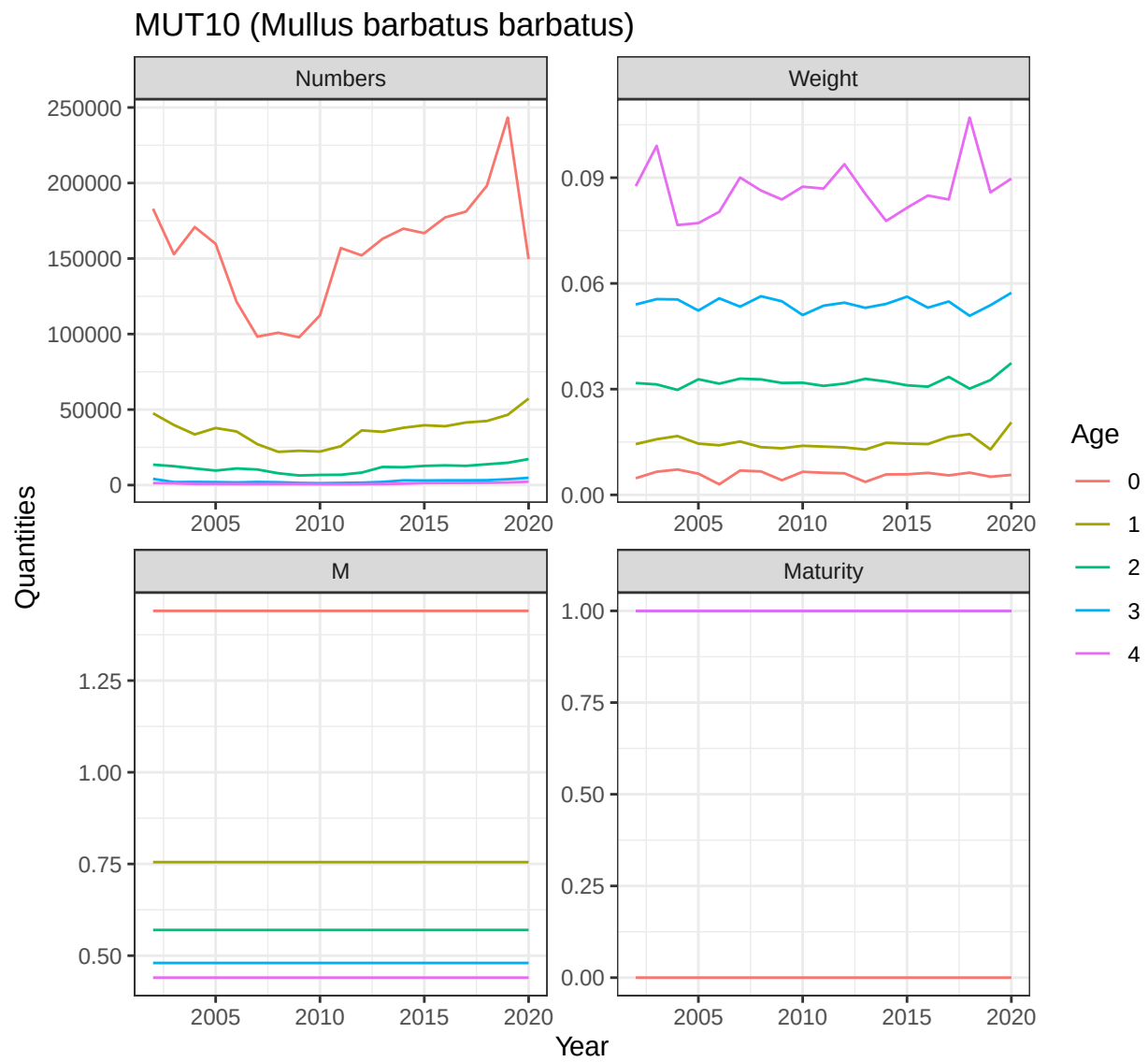


Figure 21: Stock biology trajectories at age for MUT10

```
plotbioage(stk) + ggtitle(paste0(stk@name, " (", stk@species, ")"))
```

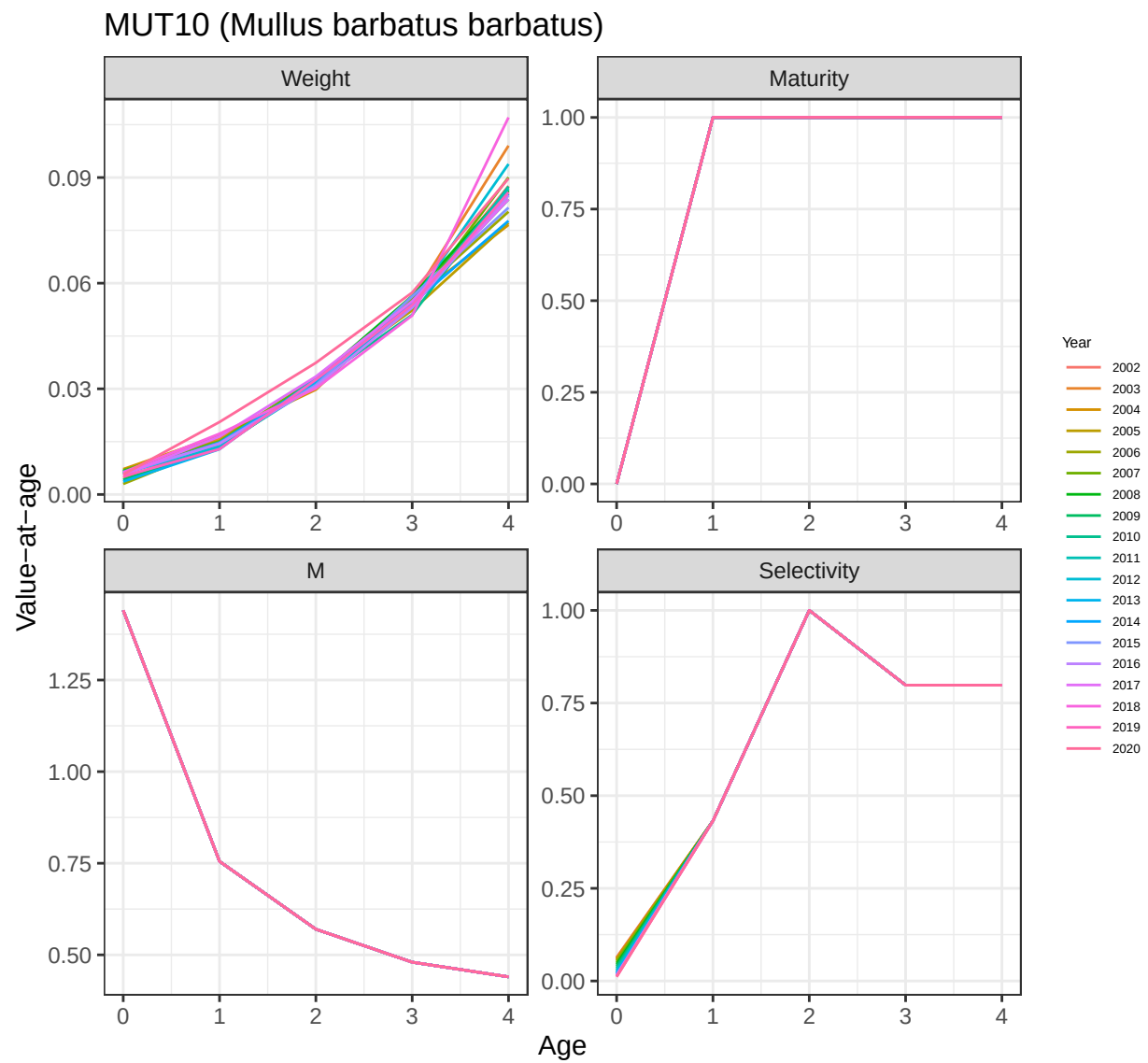


Figure 22: Annual stock quantities at age for MUT10

3.2.2 Per-recruit Analysis for MUT10

Compute the per-recruit reference points of interest

```
pr = computeFbrp(stk, proxy = "f0.1", blim = 0.25, type = "btrg", verbose = T)
Computing Per-Recruit Quantities
Computing F0.1 with Btrg = B[F.01]

Blim = 0.25 with Btrg corresponding to F0.1

# check
Fbrp(pr)
An object of class "FLPar"
params
  F0.1    Btrg    Blim    Flim    Yeq    B0
0.40140 0.00631 0.00173 2.00339 0.00255 0.01649
units:  NA
```

The following observations are noted from the below results of the per-recruit analysis for MUT10:

- The asymptotic yield curve suggests that the stock cannot be growth over-fished, mostly as a result of very high M_a and $Mat_a < Sel_a$.
- Maximum yield per recruit (YPR) is attained as F approach infinity
- As a result YPR is higher at the B_{lim} proxy of $0.25B_{F0.1}$ (approx. $0.1SPR_0$) than at $F_{0.1}$
- The age-structure is notably less truncated compared to unfished levels at $F = 0$ than observed for, e.g. MUT01.
- The current spawning potential ratio is larger than $SRP = 0.5$ and above $B_{F0.1}$

```
ploteq(pr) + ggtitle(paste0(stk@name, " (", stk@species, ")"))
```

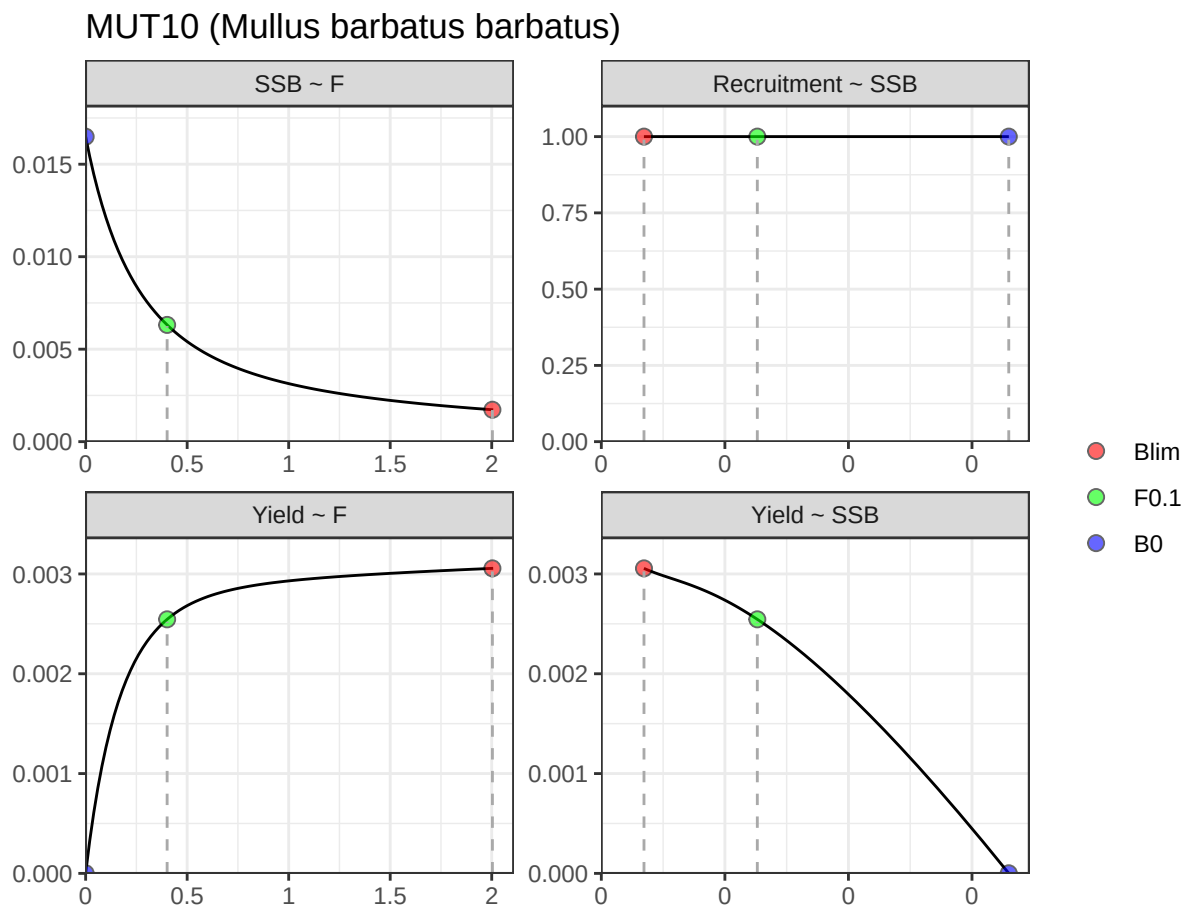


Figure 23: Per-recruit analysis for MUT10

```
plotspr(stk) + ggtitle(paste0(stk@name, " (", stk@species, ")"))
```

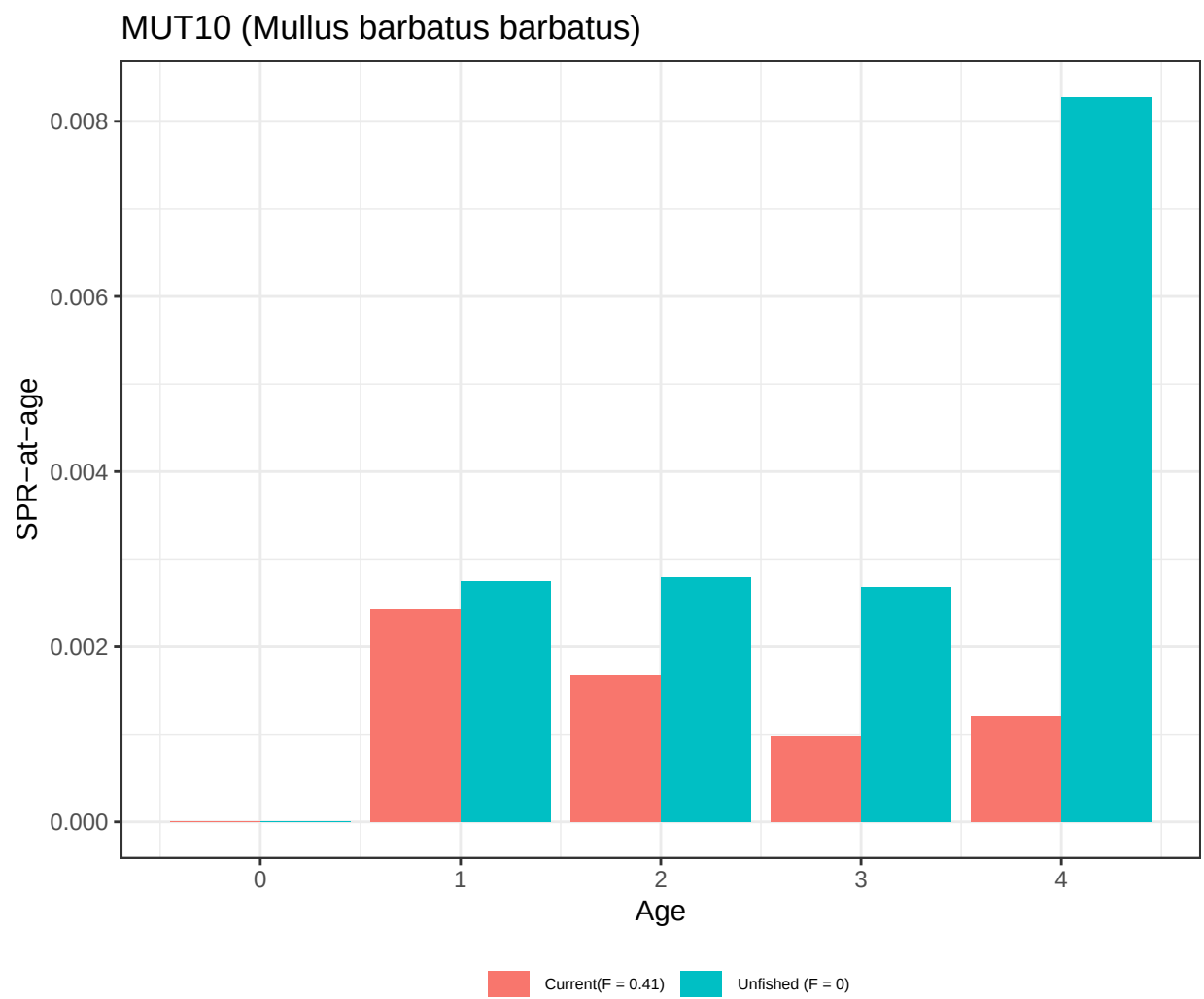


Figure 24: Comparison of the spawning biomass per recruit SPR_F at current F (average of last 3 years) and SPR_0 with $F = 0$ for MUT10

```
plotAdvice(stk, pr) + ggtitle(paste0(stk@name, " (", stk@species, ")"))
```

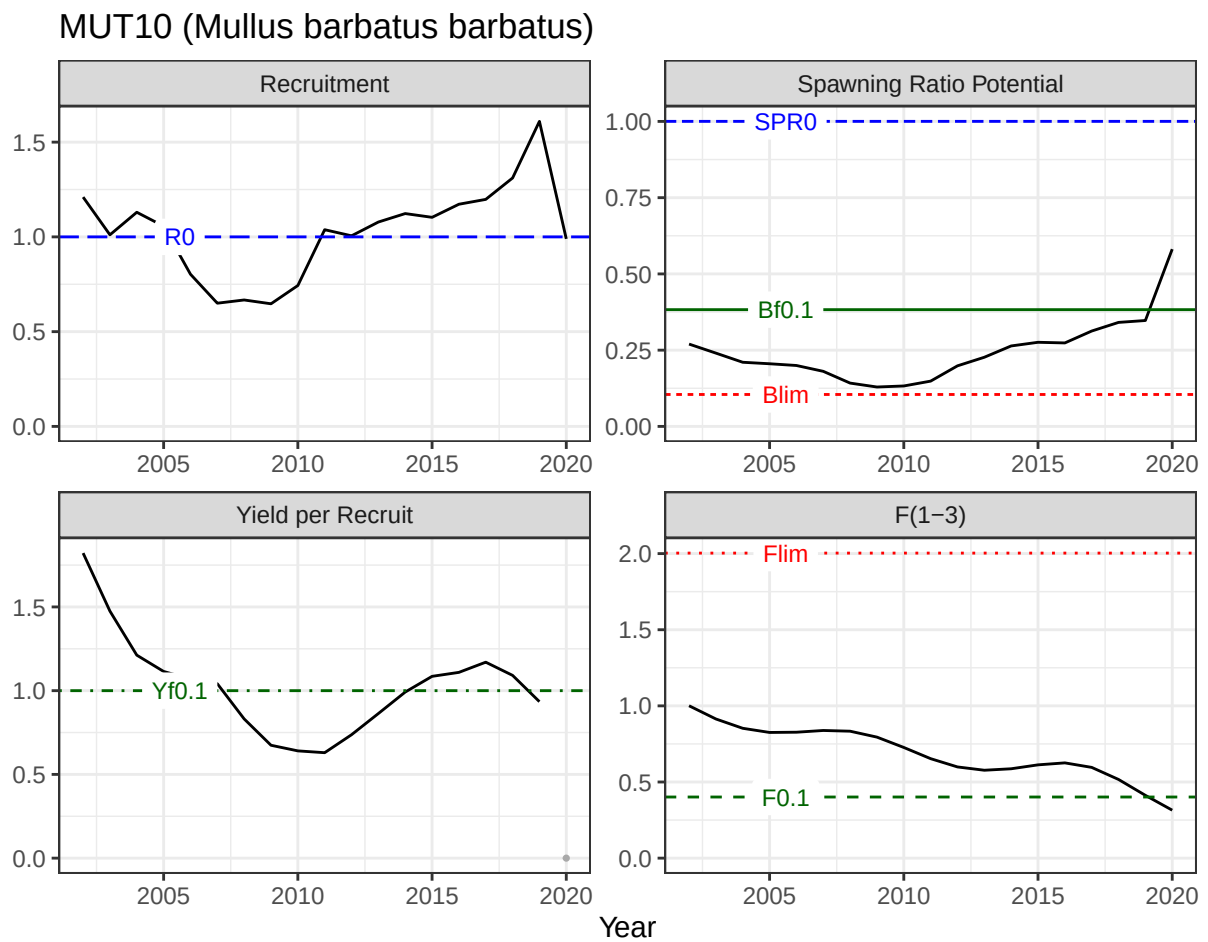


Figure 25: Time-series of stock quantities expressed as per-recruit for MUT10

3.2.3 Fitting stock-recruitment functions for MUT10

Initially four recruitment functions are explored, using the function `srrTBM` in the package `FLSRTMB`:

1. Geometric Mean (`model=geomean`)
2. Hockey-Stick (`model=segreg`)
3. Beverton-Holt (`model=bevholtSV`)
4. Ricker (`model=ricker`)

The Hockey-Stick is constrained to search for solutions of the break-point so that it falls within a lower bound (`lplim`) and upper bound (`uplim`) of spawning ration potential $SRP_{lim} = SPR_{lim}/SPR_0$. Note that in the specific case of the Hockey-Stick $SRP_{lim} = B_{lim}/B_0$, but SRP_{lim} can be generalised even if the $b = B_{lim}$ estimate is inputted into another S-R. The initial bounds are chosen to be fairly on constrained for a range of $SRP_{lim} = SRP_{0.1-20}$ by setting `lplim=0.001` and `uplim=0.2`.

In the initial fits of the Beverton-Holt and Ricker models steepness s and $R0$ are estimated given the input SPR_{0_y} .

```
# compute annual spr0y
spr0 = spr0y(stk)
# Geometric Mean
gm = srrTMB(as.FLSR(stk, model = geomean), spr0 = spr0, nyears = 3)
# Constraint Hockey-Stick
hs = srrTMB(as.FLSR(stk, model = segreg), spr0 = spr0, lplim = 0.001, uplim = 0.2,
  nyears = 3)
# Beverton Holt
bh = srrTMB(as.FLSR(stk, model = bevholtSV), spr0 = spr0, nyears = 3)
# Ricker
ri = srrTMB(as.FLSR(stk, model = rickerSV), spr0 = spr0, nyears = 3)

# Estimates
gm@SV
  s      sigmaR      R0      rho      B0
1 NA 0.2491233 151186.1 0.7277321 2493.335
hs@SV
  s      sigmaR      R0      rho      B0
1 NA 0.2424788 151186.1 0.7277321 2493.335
bh@SV
  s      sigmaR      R0      rho      B0
1 0.6673906 0.1602355 276392.4 0.2091715 4558.216
ri@SV
  s      sigmaR      R0      rho      B0
1 1.034488 0.1636943 139721.1 0.09804088 2304.256

# Combine in one object
srs = FLSRs(gm = gm, hs = hs, bh = bh, ri = ri)
```

```

p1 = plotsrs(srs)
p2 = plotsrts(srs)
p3 = plotsrs(srs, b0 = T)
p4 = plotsrs(srs, b0 = T, rel = T)

```

```

ggarrange(p1, p2, p3, p4, ncol = 2, nrow = 2, common.legend = TRUE, legend = "right")

```

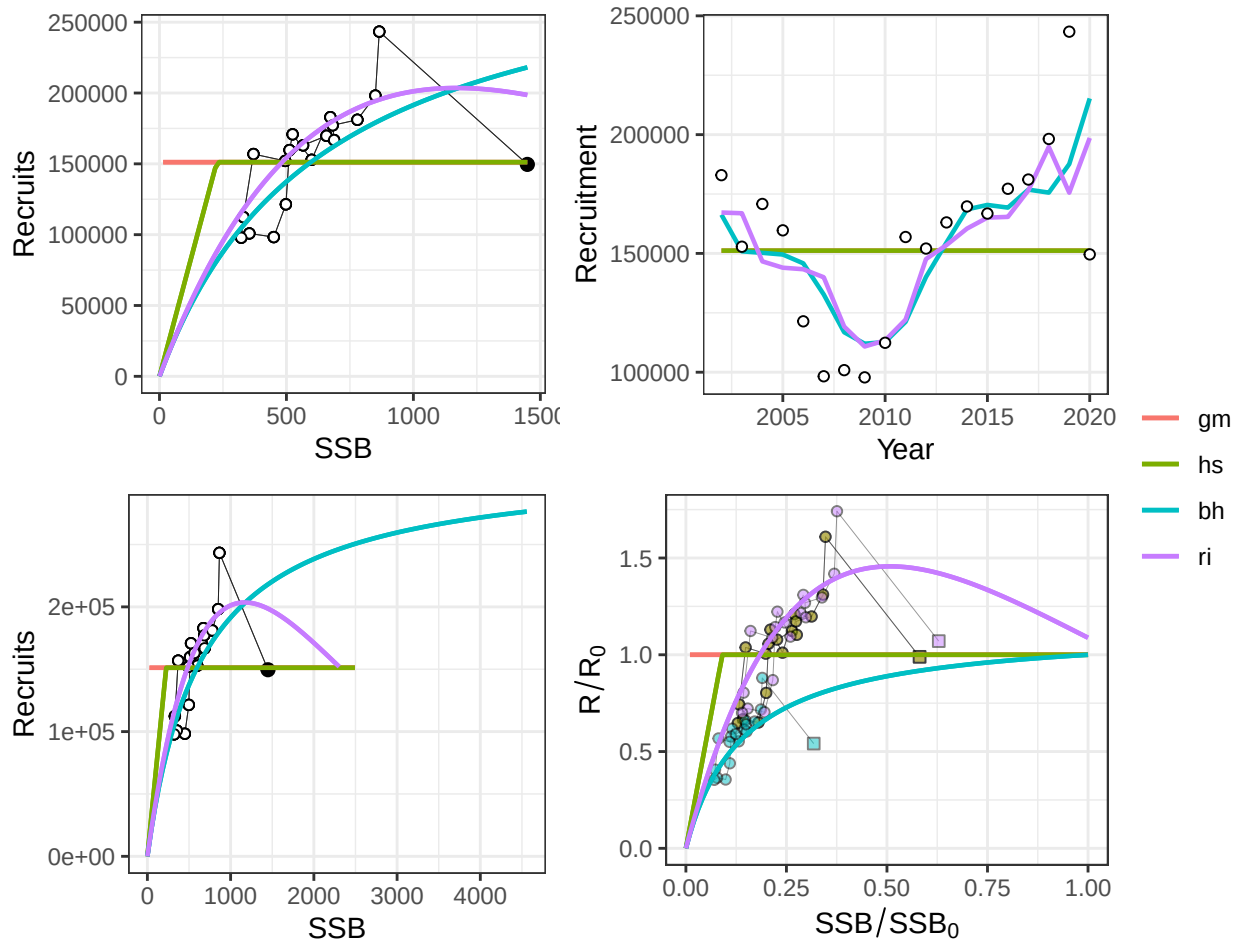


Figure 26: Fits of alternative stock-recruitment relationship for MUT10 assuming a geometric mean (gm), conditioned hockey-stick (hs), beverton-holt (bh) and ricker (ri) model

The results show that the recruitment variation is low, e.g. $\sigma_r = 0.16$ for the Beverton-Holt model, associated with a steepness of $s = 1$. The predicted recruitment by Hockey-Stick, Beverton-Holt and Ricker models follow the observed recruitment pattern over time, however, the three model differ largely in scale of their R_0 and B_0 estimates.

The break-point of the hockey-stick is estimated at $b = 226$ corresponding to an $SRP_{lim} = 0.091$ and the corresponding R_0 is the same as the geometric mean recruitment. The break-point falls outside the data and thus corresponds to the mean of the bounds `lplim` and `uplim`.

In the following the sensitivity to alternative fixed steepness values of $s = 0.8 - 0.95$ for the Beverton-Holt model explored. The results show that increasing s to 0.8-0.85 only slightly decreases the R_0 and B_0 estimates.

```
# range of s
s = seq(0.8, 0.95, 0.05)
bhs = FLSRs(lapply(s, function(x) {
  srrTMB(as.FLSR(stk, model = bevholtSV), spr0 = spr0y(stk), s.est = FALSE, s = x)
}))

# require unique names
names(bhs) = paste0("s=", s)

# Add Geomean and Hockey-Stick SRR
srs[1:2]@names
[1] "gm" "hs"

bhs = FLSRs(c(srs[1:2], bhs))
```

```
p1 = plotsrs(bhs, b0 = F)
p2 = plotsrts(bhs)
p3 = plotsrs(bhs, b0 = T)
p4 = plotsrs(bhs, b0 = T, rel = T)
ggarrange(p1, p2, p3, p4, ncol = 2, nrow = 2, common.legend = TRUE, legend = "right")
```

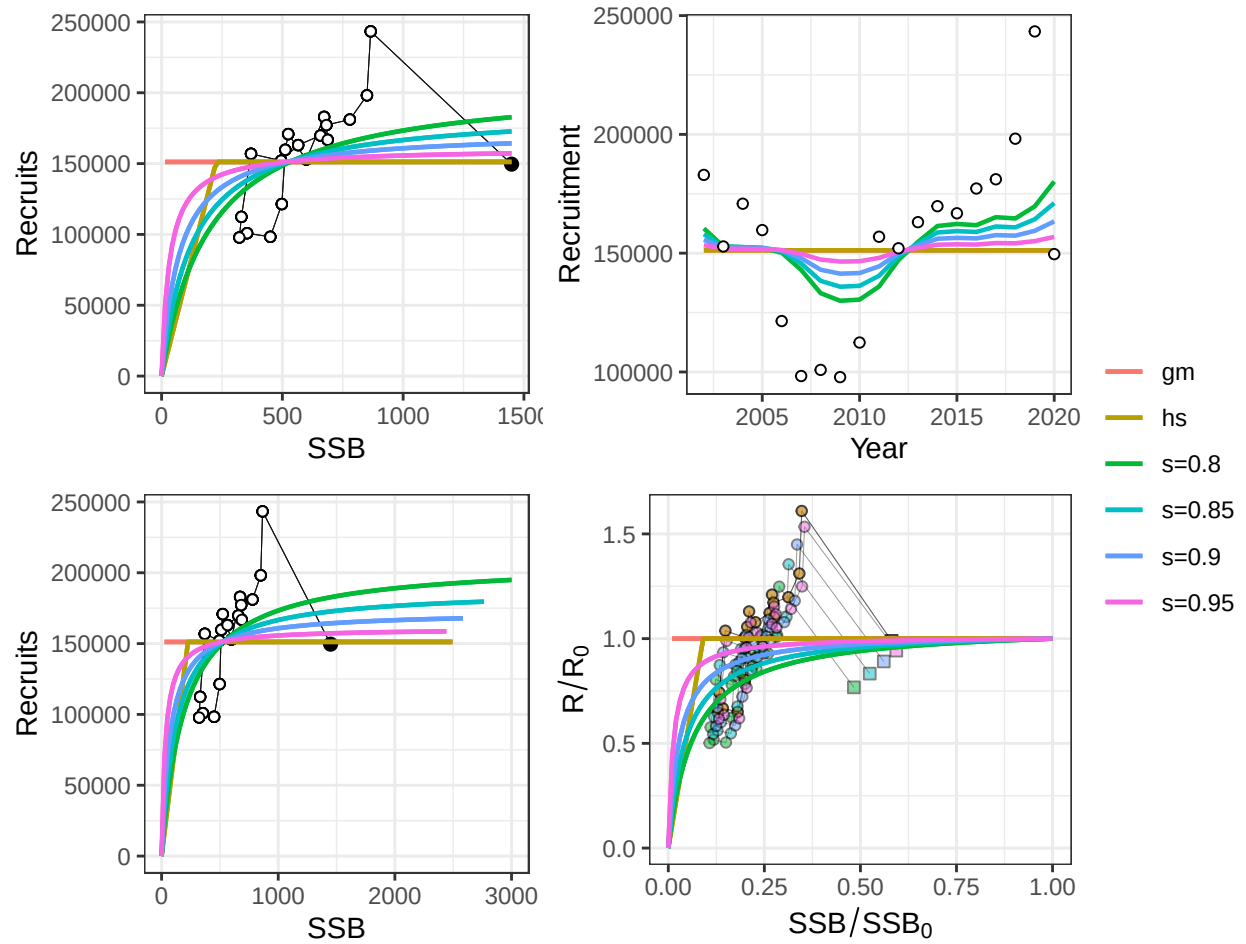


Figure 27: Fits of stock-recruitment relationships for MUT10 assuming a geometric mean (gm), conditioned hockey-stick (hs), alternative steepness s values for beverton-holt (bh) model

3.2.4 Reference point estimation for MUT10

The following candidate S-R models are considered for initial reference point estimation:

1. Hockey-Stick with B_{lim} set to the break-point $b = 226$
2. Beverton-Holt with a fixed $s = 0.85$ with $B_{lim} = 0.25B_{F0.1}$

```
# Select S-R models
selsr = FLSRs(c(srs[2], bhs[4]))
# Extract blim
blim = an(hsblim(hs)["Blim"])
# set up varying Blim parameterisation
blims = c(blim, 0.25)
types = c("value", "btrg")

brps = FLBRPs(Map(function(x, y, z) {
  computeFbrp(stk, x, proxy = "f0.1", blim = y, type = z, verbose = FALSE, fmax = 10)
}, x = selsr, y = blims, z = types))
```

```
ploteq(brps, obs = T)
Warning: Removed 4 rows containing missing values (geom_point).
```

```
ploteq(brps, obs = T, rel = T)
```

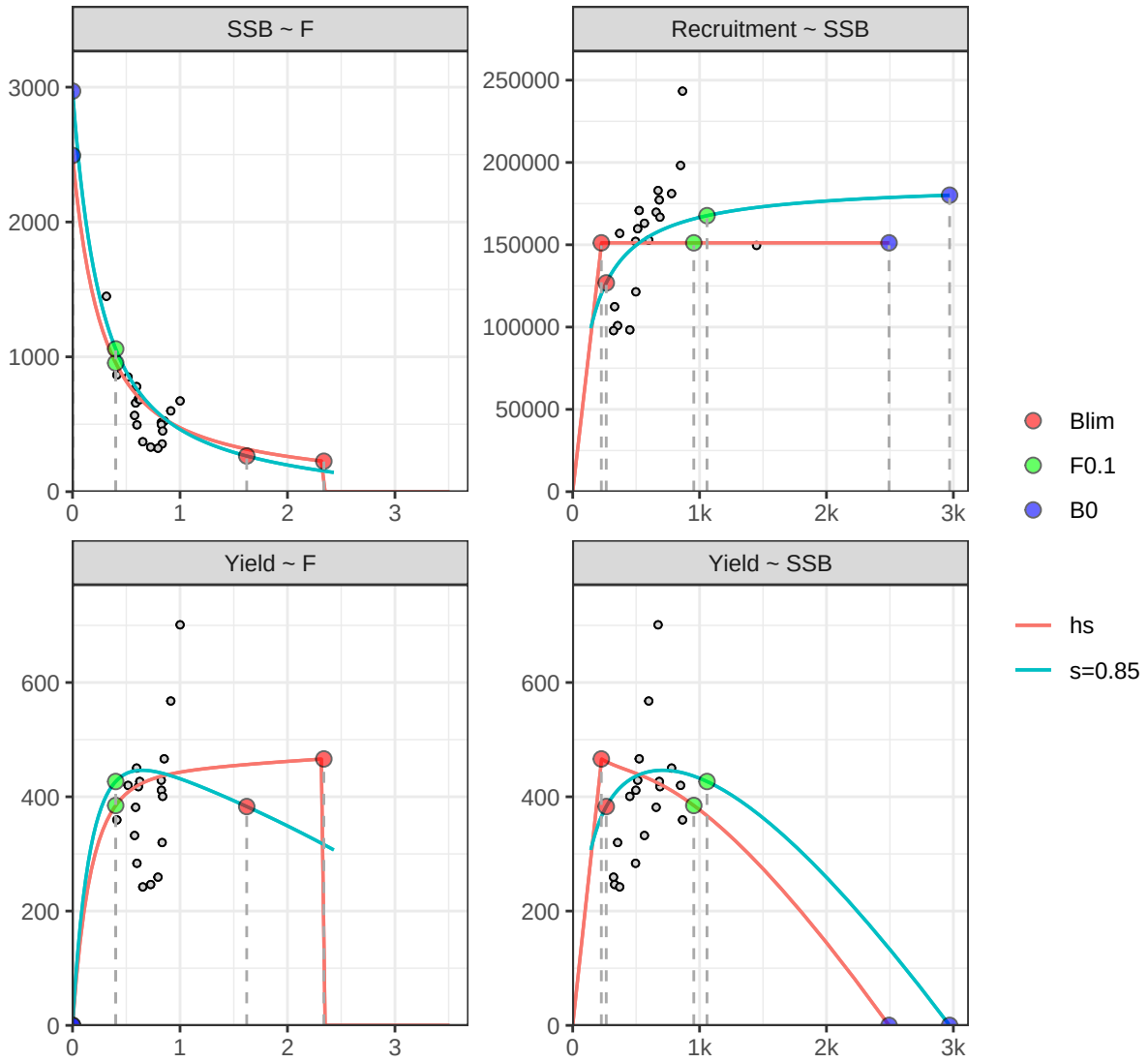


Figure 28: Reference point estimates of F_{01} , B_{lim} and B_0 shown as functions of SSB , F , Yield and Recruitment. Grey dots show the corresponding observations for MUT10

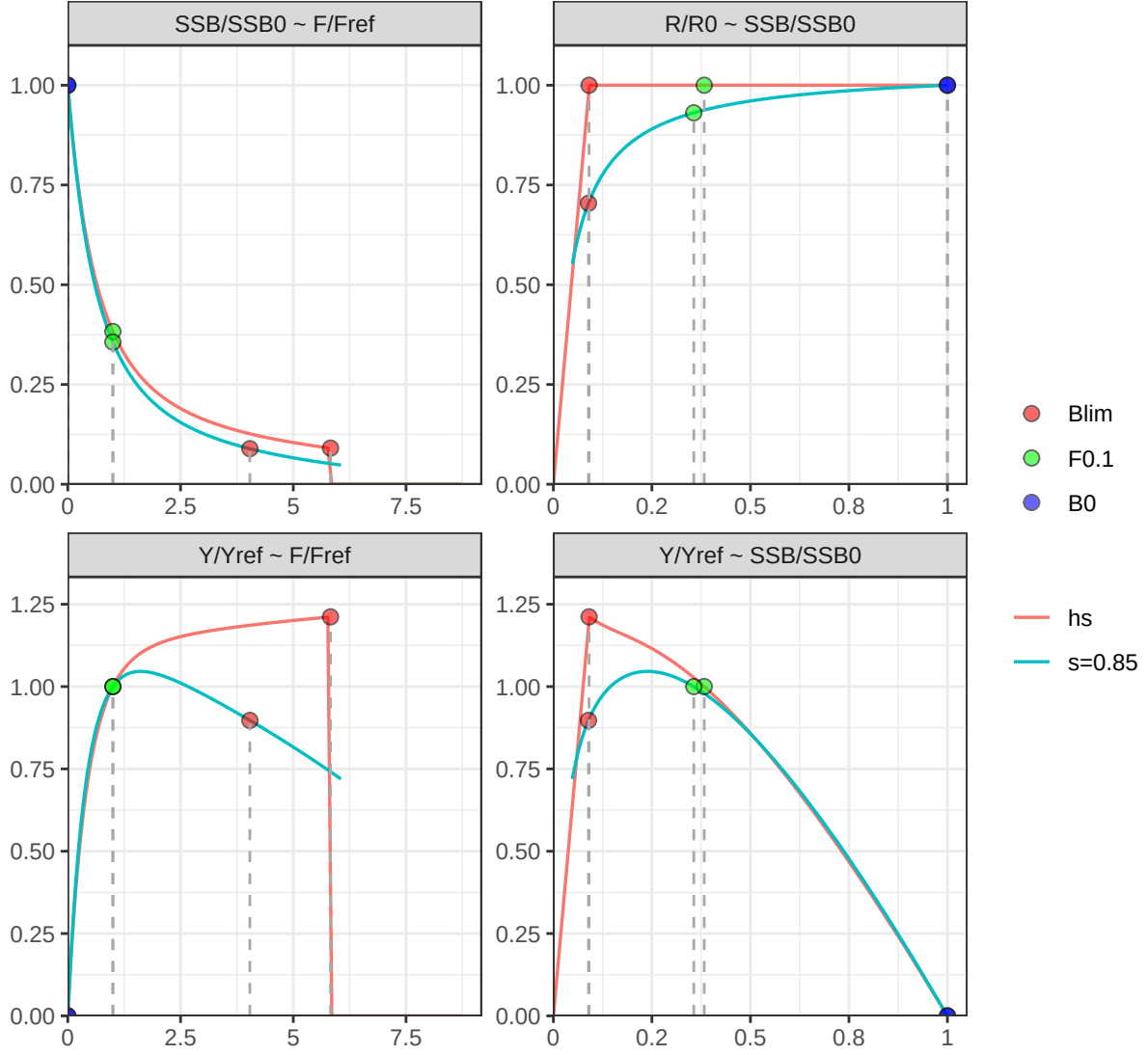


Figure 29: Relative reference point estimates of F_{01} , B_{lim} and B_0 shown as functions of SSB/SSB_0 , $F/F_{0.1}$, $Yield/Y_{F_{0.1}}$ and R/R_0 . Grey dots show the relative values of the corresponding observations for MUT10

3.2.5 Implications for stock status of MUT10

```
plotAdvice(stk, brps$hs) + ggtitle(paste0(stk@name, ": Hockey-Stick"))
```

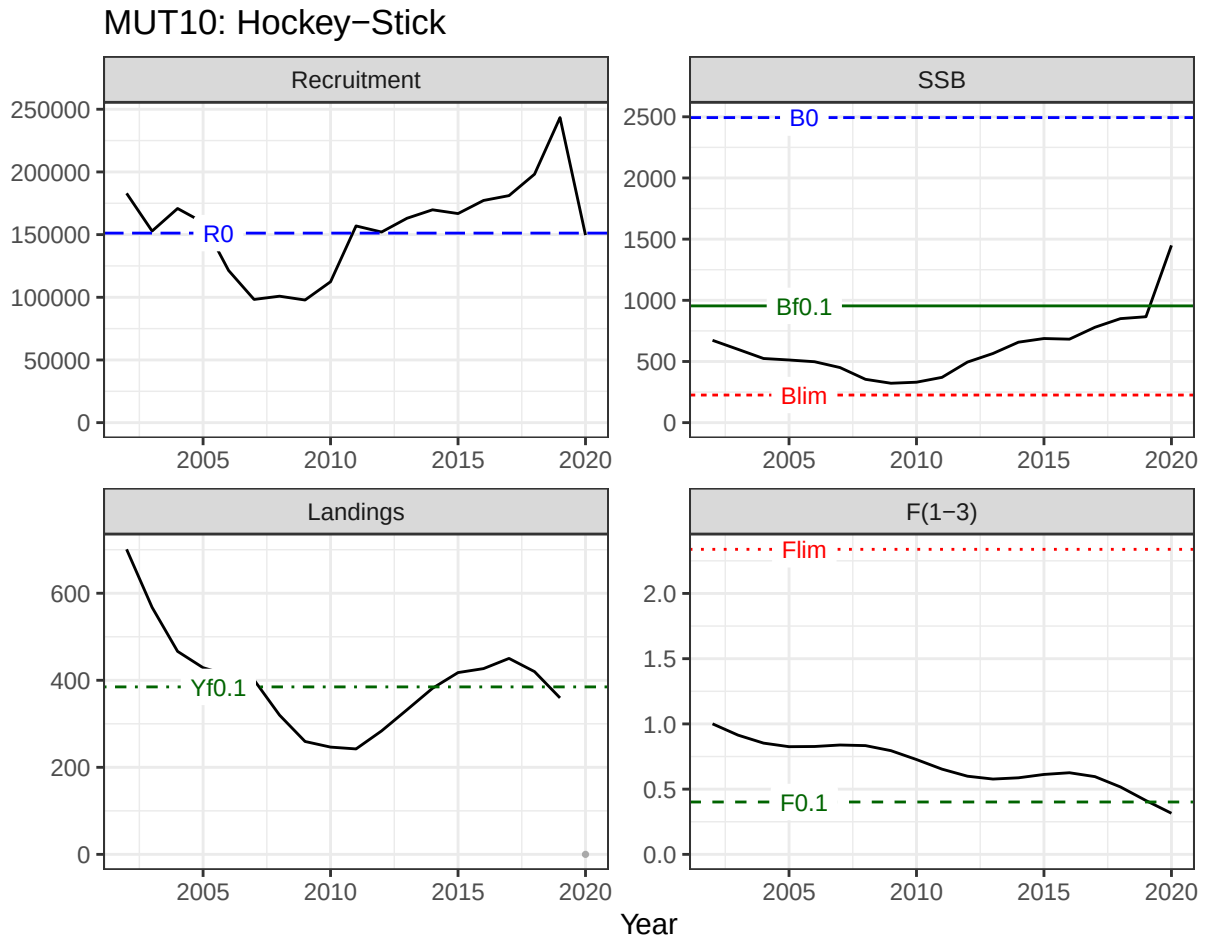


Figure 30: Status Advice plot for MUT10 showing stock trajectories of Recruitment, SSB , F , Landings and $Yield$ compared to the estimated reference points, based on a Hockey-Stick stock-recruitment relationship

```
plotAdvice(stk, brps[[2]]) + ggtitle(paste0(stk@name, ": Beverton-Holt"))
```

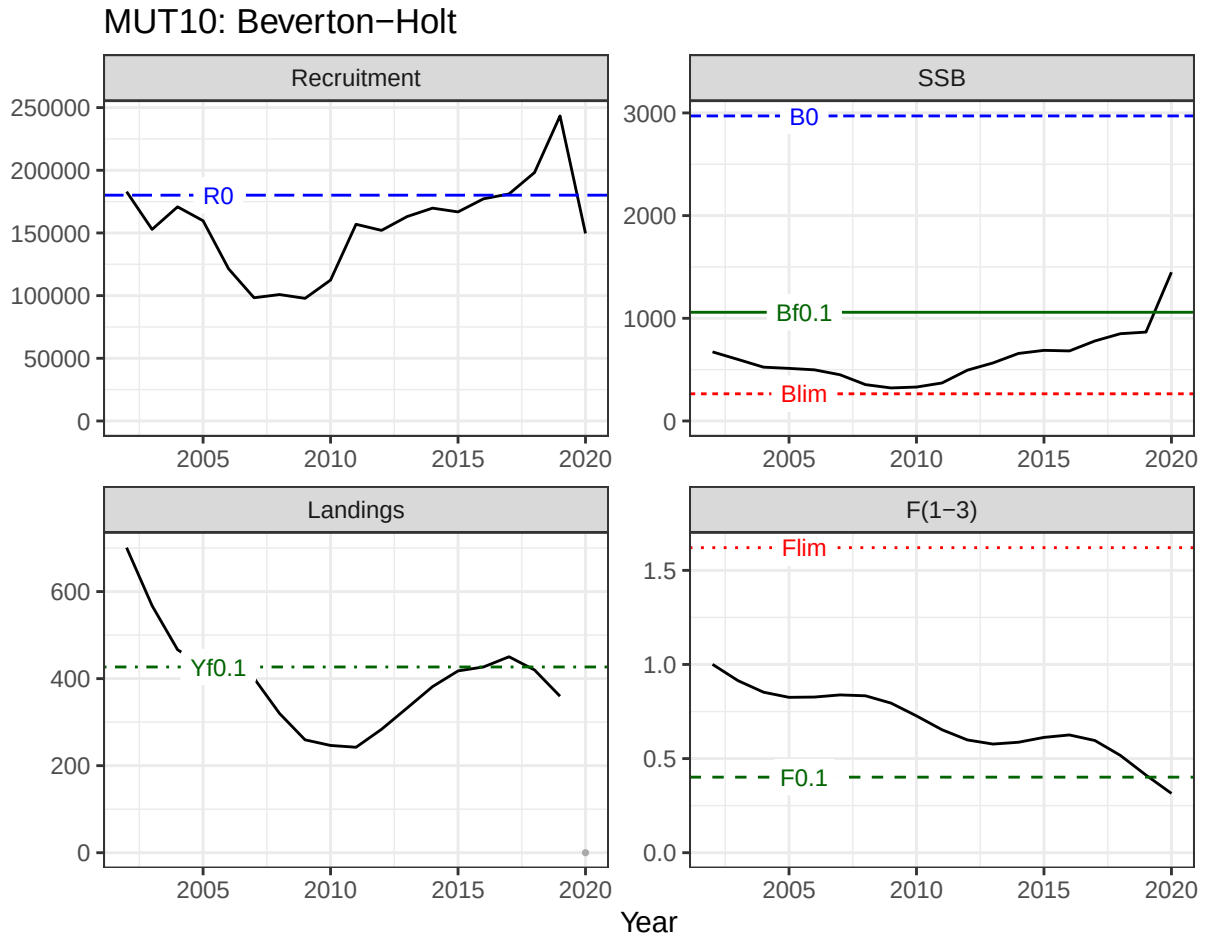


Figure 31: Status Advice plot for MUT10 showing stock trajectories of Recruitment, SSB , F , Landings and $Yield$ compared to the estimated reference points, based on a Beverton-Holt stock-recruitment relationship

```

p1 = plotAR(Fbrp(brps[[1]]), bpa = 1.4, kobe = FALSE, obs = stk) + ggtitle("a) Hockey-Stick (Absolute)")
p2 = plotAR(brps$hs, bpa = 1.4, rel = TRUE, kobe = FALSE, obs = stk) + ggtitle("b) Hockey-Stick (Relative)")
p3 = plotAR(brps[[2]], bpa = 1.4, kobe = FALSE, obs = stk) + ggtitle("c) Beverton-Holt (Absolute)")
p4 = plotAR(brps[[2]], bpa = 1.4, rel = TRUE, kobe = FALSE, obs = stk) + ggtitle("d) Beverton-Holt (Relative)")
ggarrange(p1, p2, p3, p4)

```

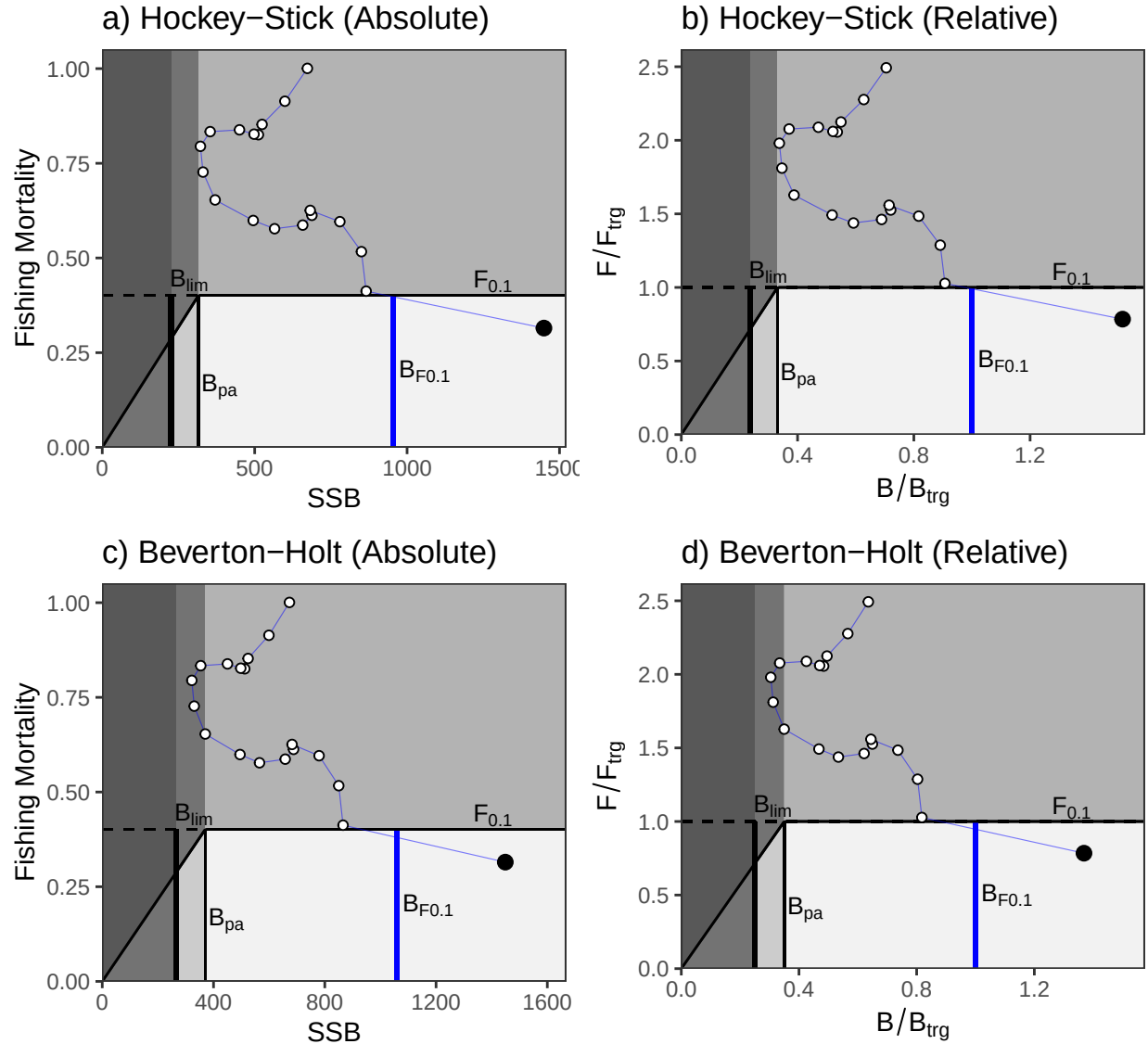


Figure 32: Advice Rule plots for MUT10, with $B_{pa} = 1.4B_{lim}$, showing a comparison between a Hockey-Stick and Beverton-Holt S-R with an empirical $B_{lim} = 0.25B_{F0.1}$.

4 Hake

Subset hake stocks

4.1 HKE01.05.06.07

4.1.1 Stock dynamics of HKE01.05.06.07

Notable features of the HKE01.05.06.07 stock are: + Selectivity exhibits some extreme dynamics over the period 2008-2013 + F_a for age-4 peaked above 7.5! in 2011 with severe knockout effects on the plus group + High M_a at age-0 and age-1, which also contribute most to the currenence biomass + Biomass largely driven by ages 1-2. + In recent years the models predicts that mostly ages 1-3 are effectively selected + This would make HKE01.05.06.07 likely vunarable to growth overfishing.

```
plot(stk) + theme_bw() + facet_wrap(~qname, scales = "free", ncol = 2) + ggtitle(paste0(stk@name,
  " (", stk@species, ")"))
```

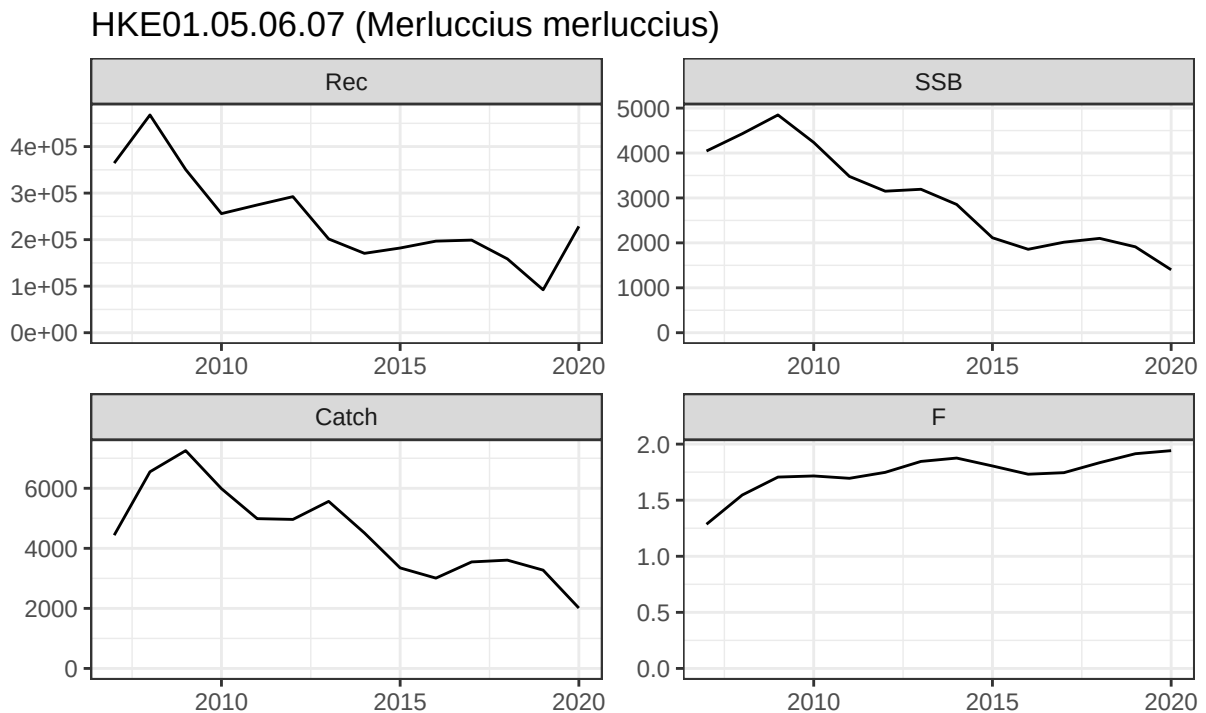


Figure 33: Estimated stock assessment trajectories for HKE01.05.06.07

```
plotdyn(stk) + ggtitle(paste0(stk$name, " (", stk$species, ")"))
```

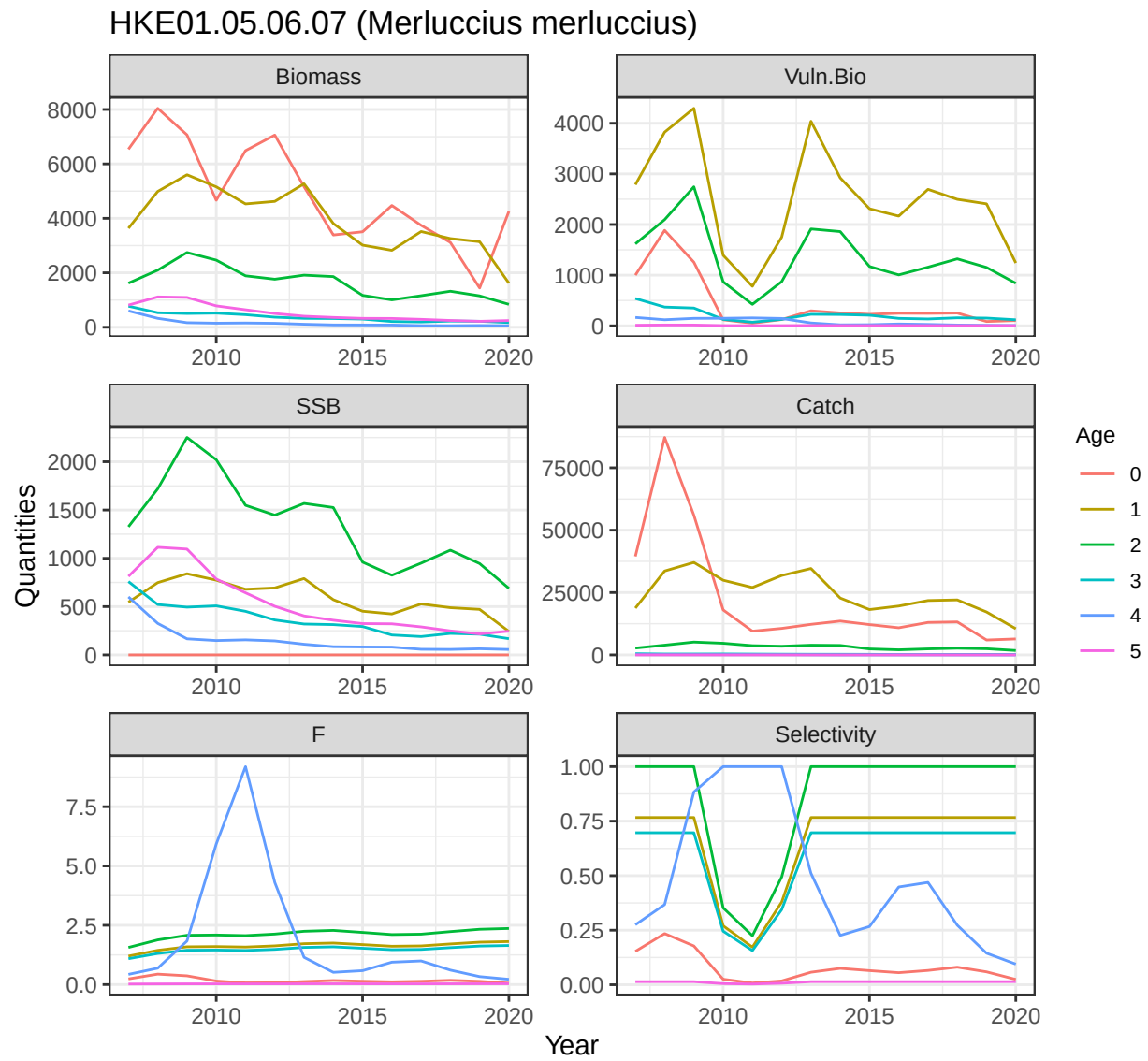


Figure 34: Stock assessment trajectories at age for HKE01.05.06.07

```
plotbiogr(stk) + ggtitle(paste0(stk@name, " (", stk@species, ")"))
```



Figure 35: Stock biology trajectories at age for HKE01.05.06.07

```
plotbioage(stk) + ggtitle(paste0(stk@name, " (", stk@species, ")"))
```

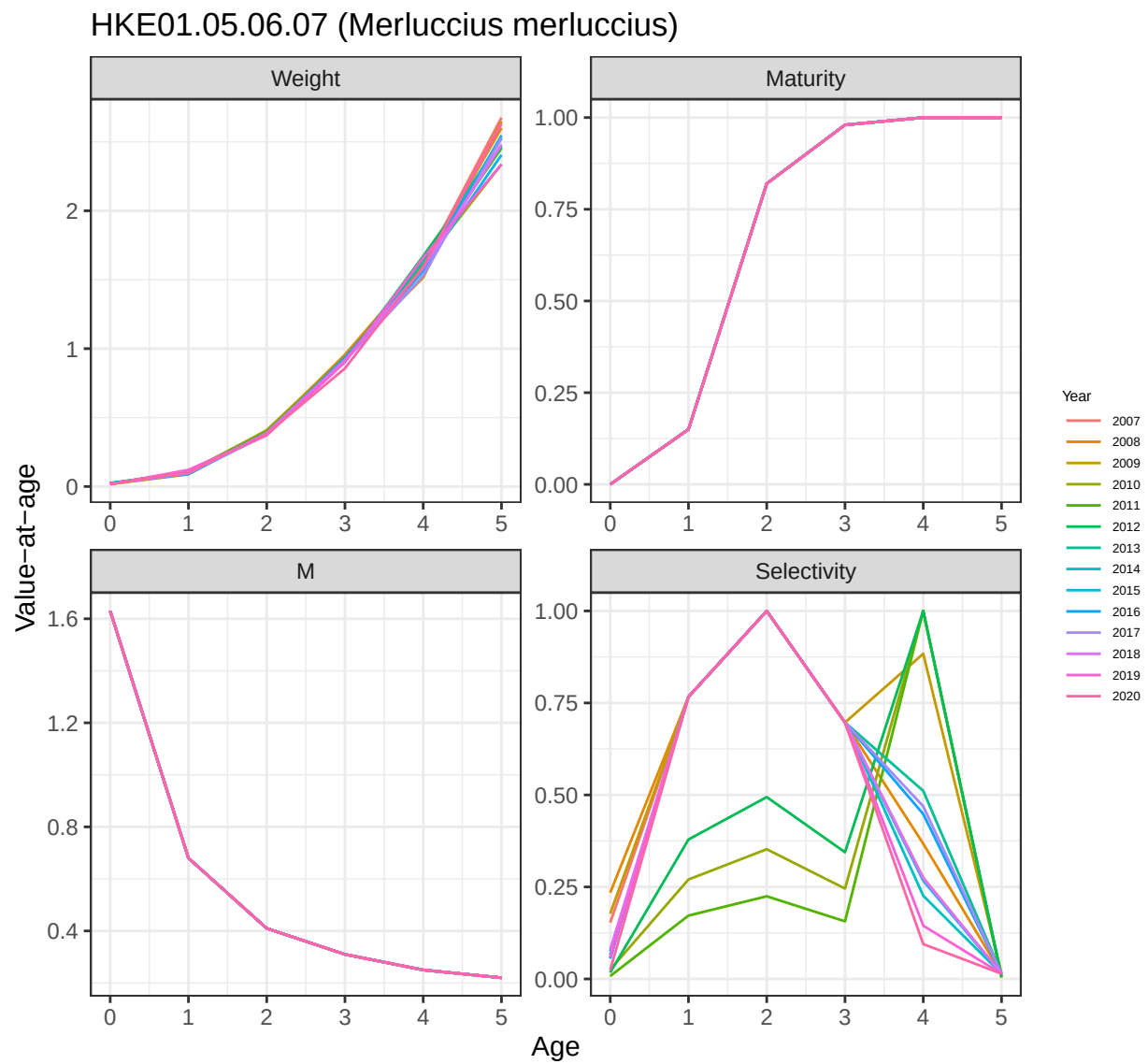


Figure 36: Annual stock quantities at ageHKE01.05.06.07

4.1.2 Per-recruit Analysis for HKE01.05.06.07

Compute the per-recruit reference points of interest

```
pr = computeFbrp(stk, proxy = "f0.1", blim = 0.25, type = "btrg", verbose = T)
Computing Per-Recruit Quantities
Computing F0.1 with Btrg = B[F.01]

Blim = 0.25 with Btrg corresponding to F0.1

# check
Fbrp(pr)
An object of class "FLPar"
params
  F0.1   Btrg   Blim   Flim   Yeq   B0
0.4445 0.1714 0.0428 0.9801 0.0221 0.6427
units:  NA
```

The following observations can be noted from the below results of the per-recruit analysis for HKE01.05.06.07:

- The yield curve shows some curvature indicating some degree of growth overfishing
- $F_{0.1}$ is relatively close to the maximum along the fairly flat top of the yield curve
- YPR at $F_{0.1}$ is comparable to the YPR at the B_{lim} proxy of $0.25B_{F0.1}$
- The age-structure is extremely truncated to unfished levels at $F = 0$ and virtually non-existent for ages 3+
- The corresponding spawning potential ratio less than $SRP = 0.05$ falls below the B_{lim} proxy.

```
ploteq(pr) + ggtitle(paste0(stk@name, " (", stk@species, ")"))
```

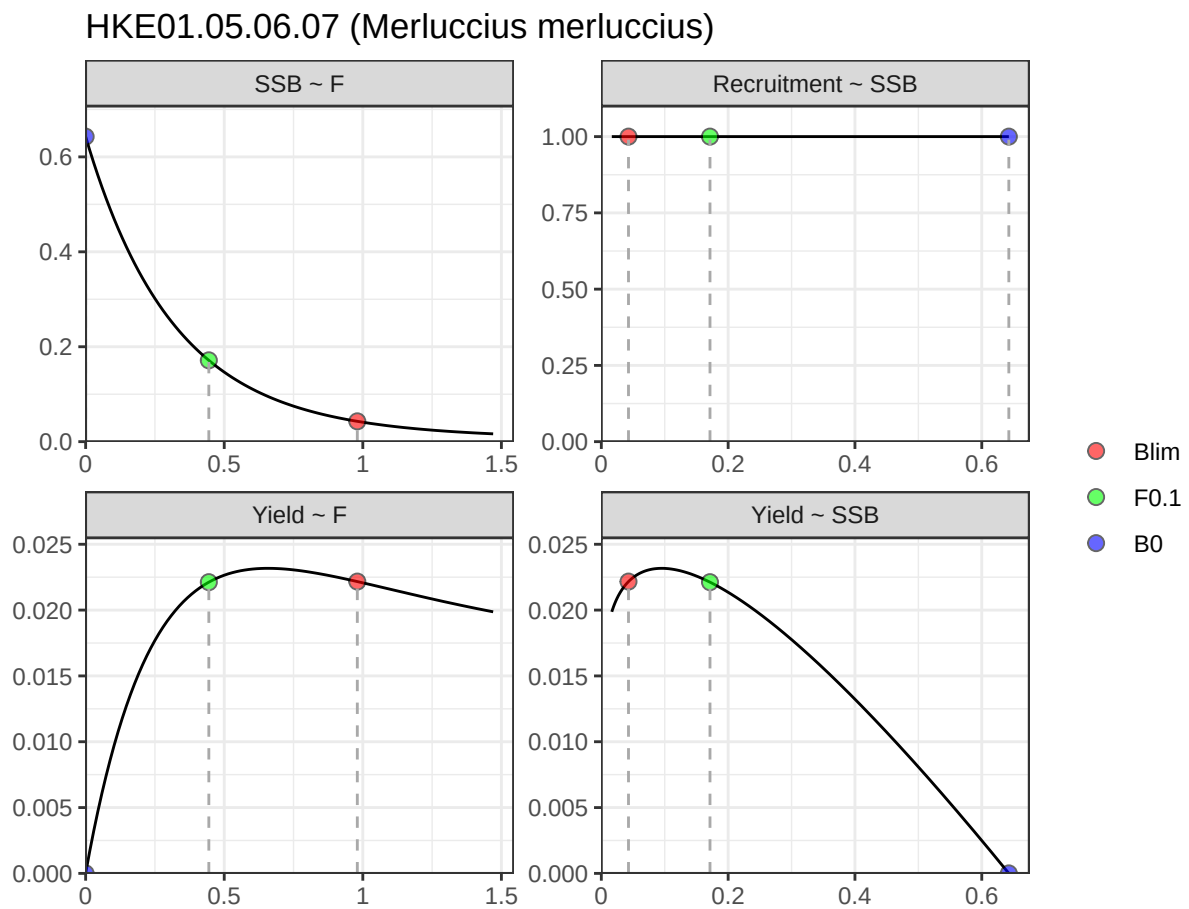


Figure 37: Per-recruit analysis for HKE01.05.06.07

```
plotspr(stk) + ggtitle(paste0(stk@name, " (", stk@species, ")"))
```

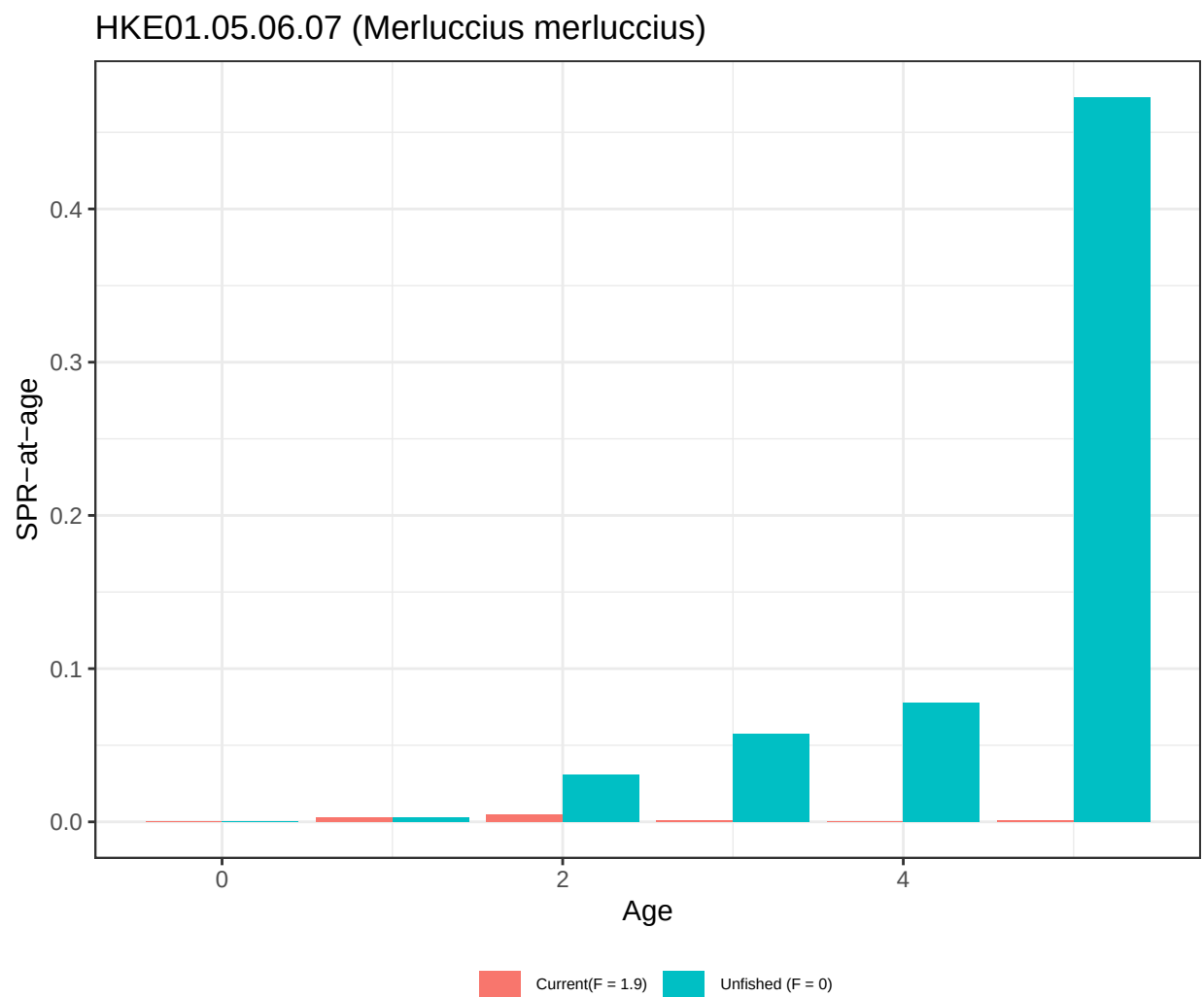


Figure 38: Comparison of the spawning biomass per recruit SPR_F at current F (average of last 3 years) and SPR_0 with $F = 0$ for HKE01.05.06.07

```
plotAdvice(stk, pr) + ggtitle(paste0(stk@name, " (", stk@species, ")"))
```

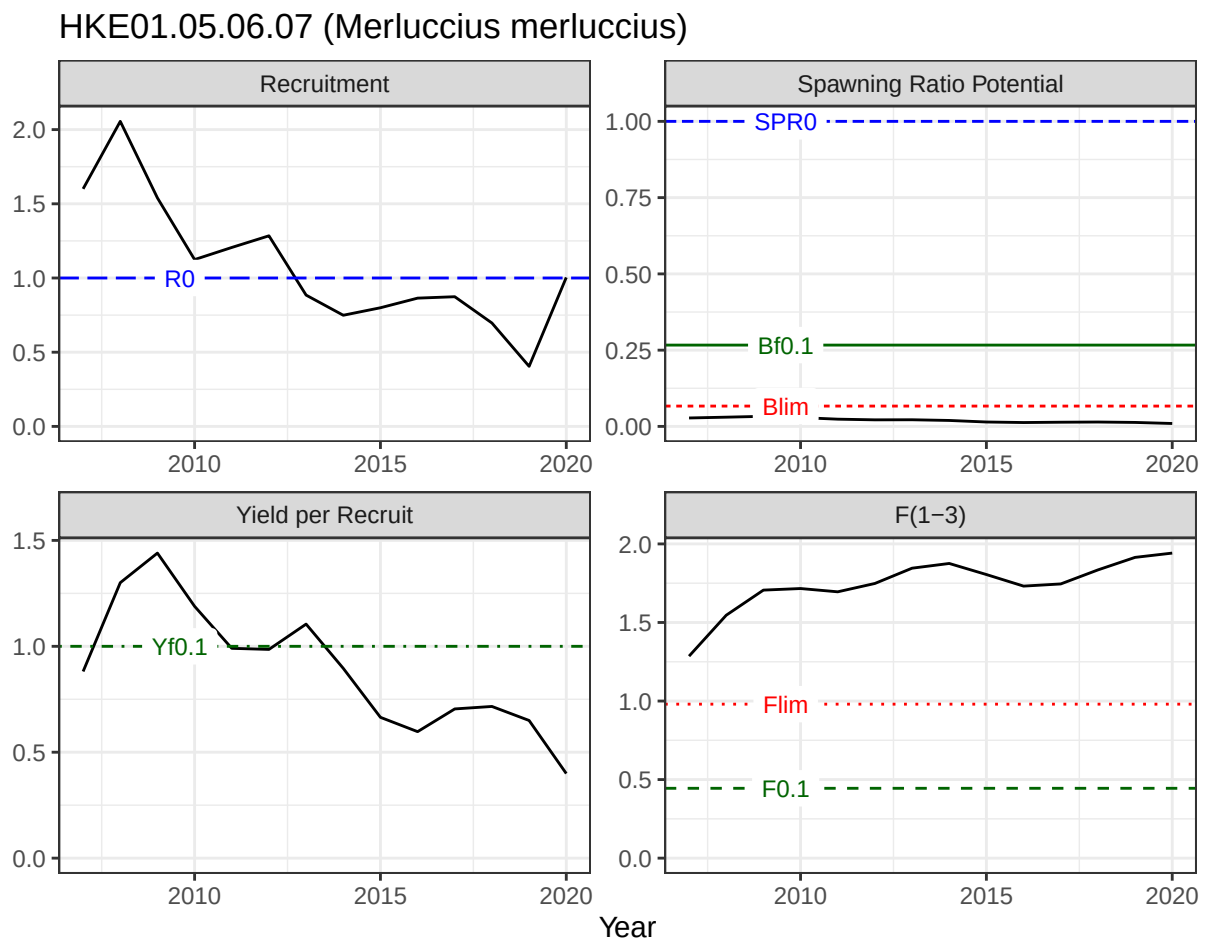


Figure 39: Time-series of stock quantaties expressed as per-recruit for HKE01.05.06.07

4.1.3 Fitting stock-recruitment functions for HKE01.05.06.07

Initially four recruitment functions are explored, using the function `srrTBM` in the package `FLSRTMB`:

1. Geometric Mean (`model=geomean`)
2. Hockey-Stick (`model=segreg`)
3. Beverton-Holt (`model=bevholtSV`)
4. Ricker (`model=ricker`)

The Hockey-Stick is constrained to search for solutions of the break-point so that it falls within a lower bound (`lplim`) and upper bound (`uplim`) of spawning ration potential $SRP_{lim} = SPR_{lim}/SPR_0$. Note that in the specific case of the Hockey-Stick $SRP_{lim} = B_{lim}/B_0$, but SRP_{lim} can be generalised even if the $b = B_{lim}$ estimate is inputted into another S-R. The initial bounds are chosen to be fairly on constrained for a range of $SRP_{lim} = SRP_{0.1-20}$ by setting `lplim=0.001` and `uplim=0.2`.

In the initial fits of the Beverton-Holt and Ricker models steepness s and $R0$ are estimated given the input SPR_{0_y} .

```
# compute annual spr0y
spr0 = spr0y(stk)
# Geometric Mean
gm = srrTMB(as.FLSR(stk, model = geomean), spr0 = spr0, nyears = 3)
# Constraint Hockey-Stick
hs = srrTMB(as.FLSR(stk, model = segreg), spr0 = spr0, lplim = 0.001, uplim = 0.2,
  nyears = 3)
# Beverton Holt
bh = srrTMB(as.FLSR(stk, model = bevholtSV), spr0 = spr0, nyears = 3)
# Ricker
ri = srrTMB(as.FLSR(stk, model = rickerSV), spr0 = spr0, nyears = 3)

# Estimates
gm@SV
  s      sigmaR      R0      rho      B0
1 NA 0.4089106 227679.5 0.6155217 146330.5
hs@SV
  s      sigmaR      R0      rho      B0
1 NA 0.3090826 347580.1 -0.2197609 223391.1
bh@SV
  s      sigmaR      R0      rho      B0
1 0.9472421 0.3009181 910966.2 -0.1220996 585481.4
ri@SV
  s      sigmaR      R0      rho      B0
1 5.624471 0.2836674 92203.17 -0.1775855 59259.33

# Combine in one object
srs = FLSRs(gm = gm, hs = hs, bh = bh, ri = ri)
```

```

p1 = plotsrs(srs)
p2 = plotsrts(srs)
p3 = plotsrs(srs, b0 = T)
p4 = plotsrs(srs, b0 = T, rel = T)

```

```

ggarrange(p1, p2, p3, p4, ncol = 2, nrow = 2, common.legend = TRUE, legend = "right")

```

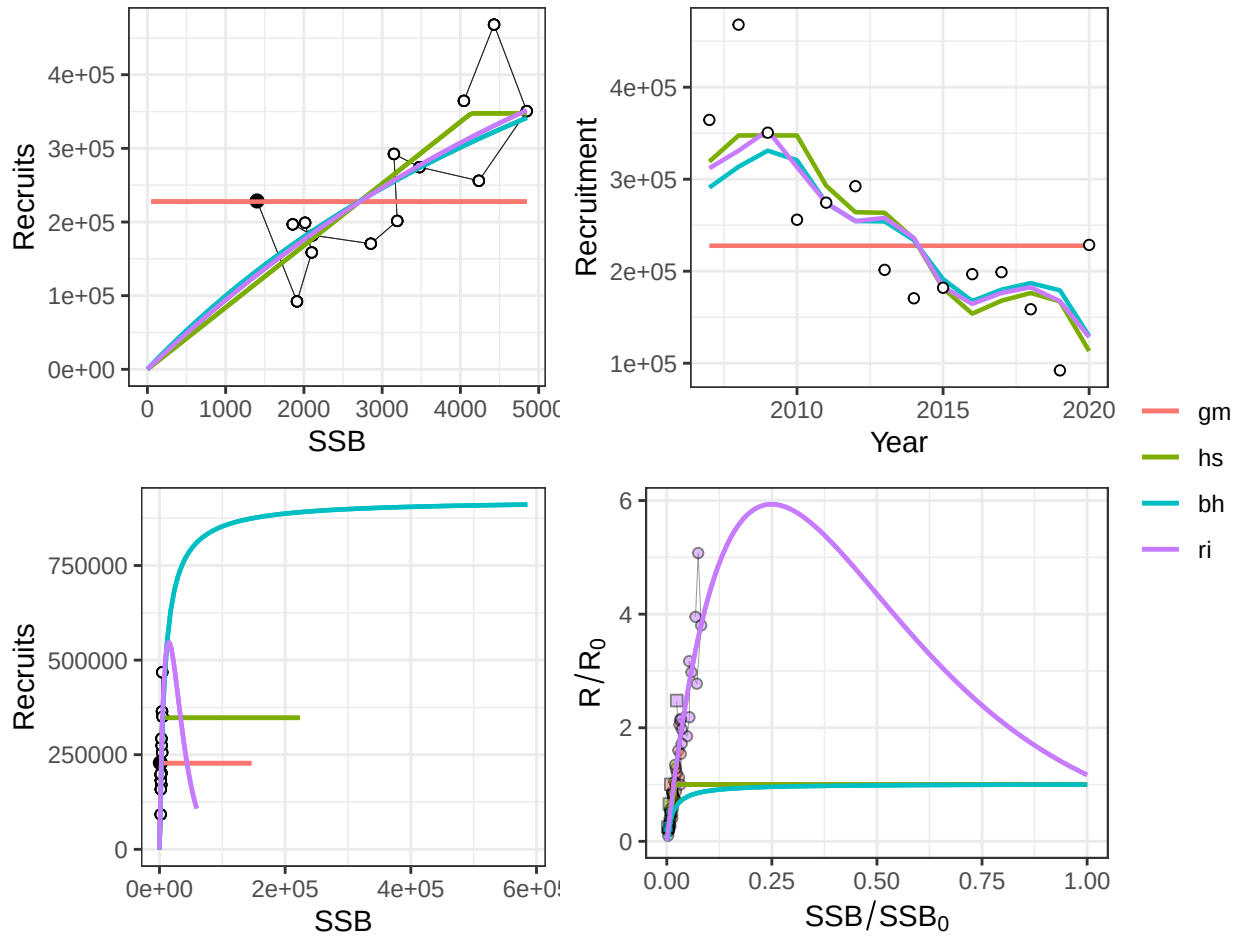


Figure 40: Fits of alternative stock-recruitment relationship for HKE01.05.06.07 assuming a geometric mean (gm), conditioned hockey-stick (hs), beverton-holt (bh) and ricker (ri) model

The results show that the recruitment variation is moderately low, e.g. $\sigma_r = 0.3$ for the Beverton-Holt model, associated with a steepness $s = 0.95$. The predicted recruitment by Hockey-Stick, Beverton-Holt and Ricker models follow the observed recruitment pattern over time, however, the three model differ largely in scale of their R_0 and B_0 estimates.

The break-point of the hockey-stick is estimated at $b = 4138$ corresponding to an $SRP_{lim} = 0.019$ and the corresponding R_0 is slightly above the geometric mean recruitment.

In the following the sensitivity to alternative fixed steepness values of $s = 0.8 - 0.95$ for the Beverton-Holt model explored. The results show that only increasing $s = 0.95$ to 0.95 can substantially decrease the R_0 and B_0 estimates to a scale that is more comparable to the Hockey-Stick estimates.

```
# range of s
s = seq(0.8, 0.95, 0.05)
bhs = FLSRs(lapply(s, function(x) {
  srrTMB(as.FLSR(stk, model = bevholtSV), spr0 = spr0y(stk), s.est = FALSE, s = x)
}))

# require unique names
names(bhs) = paste0("s=", s)

# Add Geomean and Hockey-Stick SRR
srs[1:2]@names
[1] "gm" "hs"

bhs = FLSRs(c(srs[1:2], bhs))
```

```
p1 = plotsrs(bhs, b0 = F)
p2 = plotsrts(bhs)
p3 = plotsrs(bhs, b0 = T)
p4 = plotsrs(bhs, b0 = T, rel = T)
ggarrange(p1, p2, p3, p4, ncol = 2, nrow = 2, common.legend = TRUE, legend = "right")
```

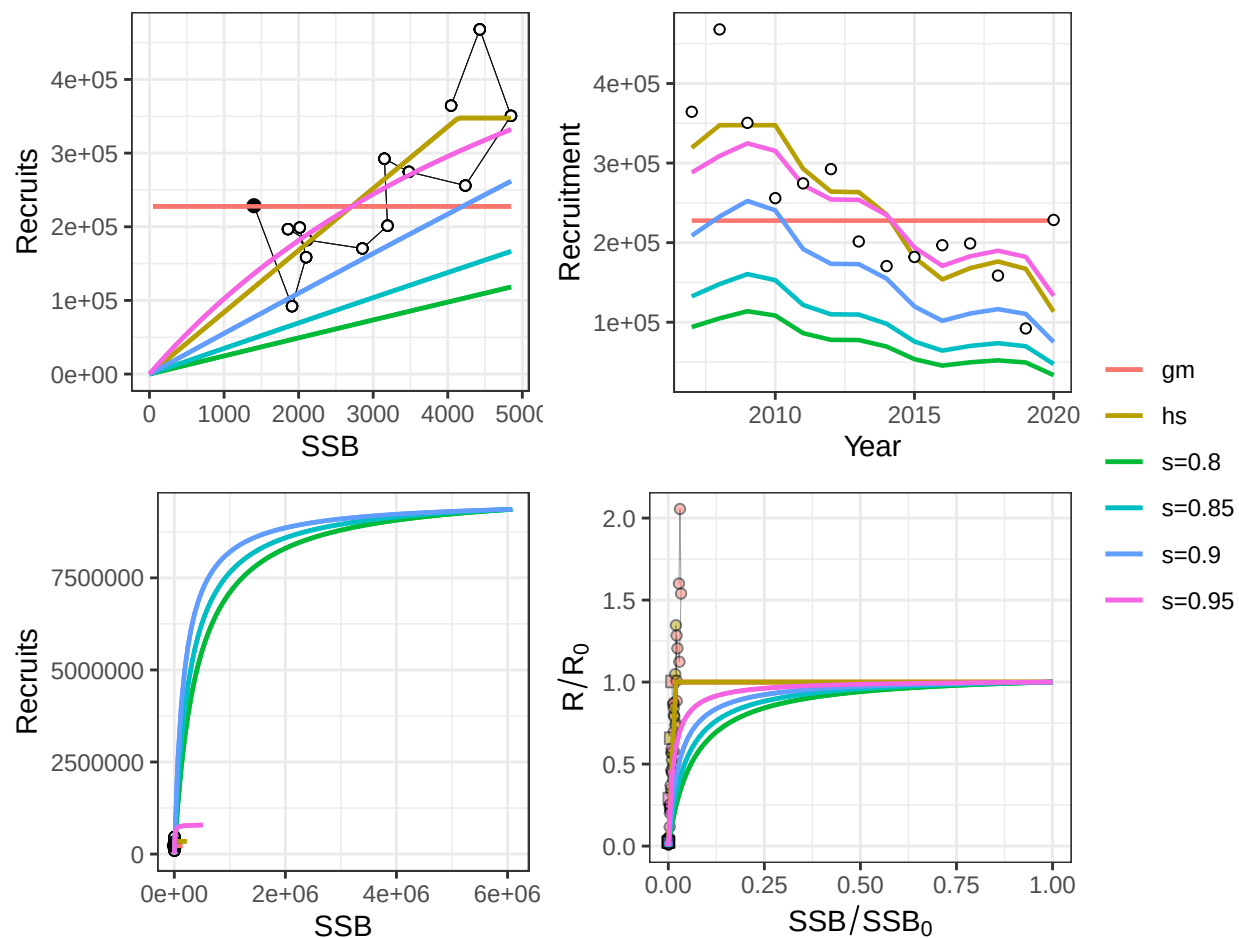


Figure 41: Fits of stock-recruitment relationships for HKE01.05.06.07 assuming a geometric mean (gm), conditioned hockey-stick (hs), alternative steepness s values for beverton-holt (bh) model

4.1.4 Reference point estimation for HKE01.05.06.07

The following candidate S-R models are considered for initial reference point estimation:

1. Hockey-Stick with B_{lim} set to the break-point $b = 4138$
2. Beverton-Holt with a fixed $s = 0.95$ with $B_{lim} = 0.25B_{F0.1}$ ⁴

```
# Select S-R models
selsr = FLSRs(c(srs[2], bhs[6]))
# Extract blim
blim = an(hsblim(hs)["Blim"])
# set up varying Blim parameterisation
blims = c(blim, 0.25)
types = c("value", "btrg")

brps = FLBRPs(Map(function(x, y, z) {
  computeFbrp(stk, x, proxy = "f0.1", blim = y, type = z, verbose = FALSE, fmax = 10)
}, x = selsr, y = blims, z = types))
```

```
ploteq(brps, obs = T)
```

```
ploteq(brps, obs = T, rel = T)
```

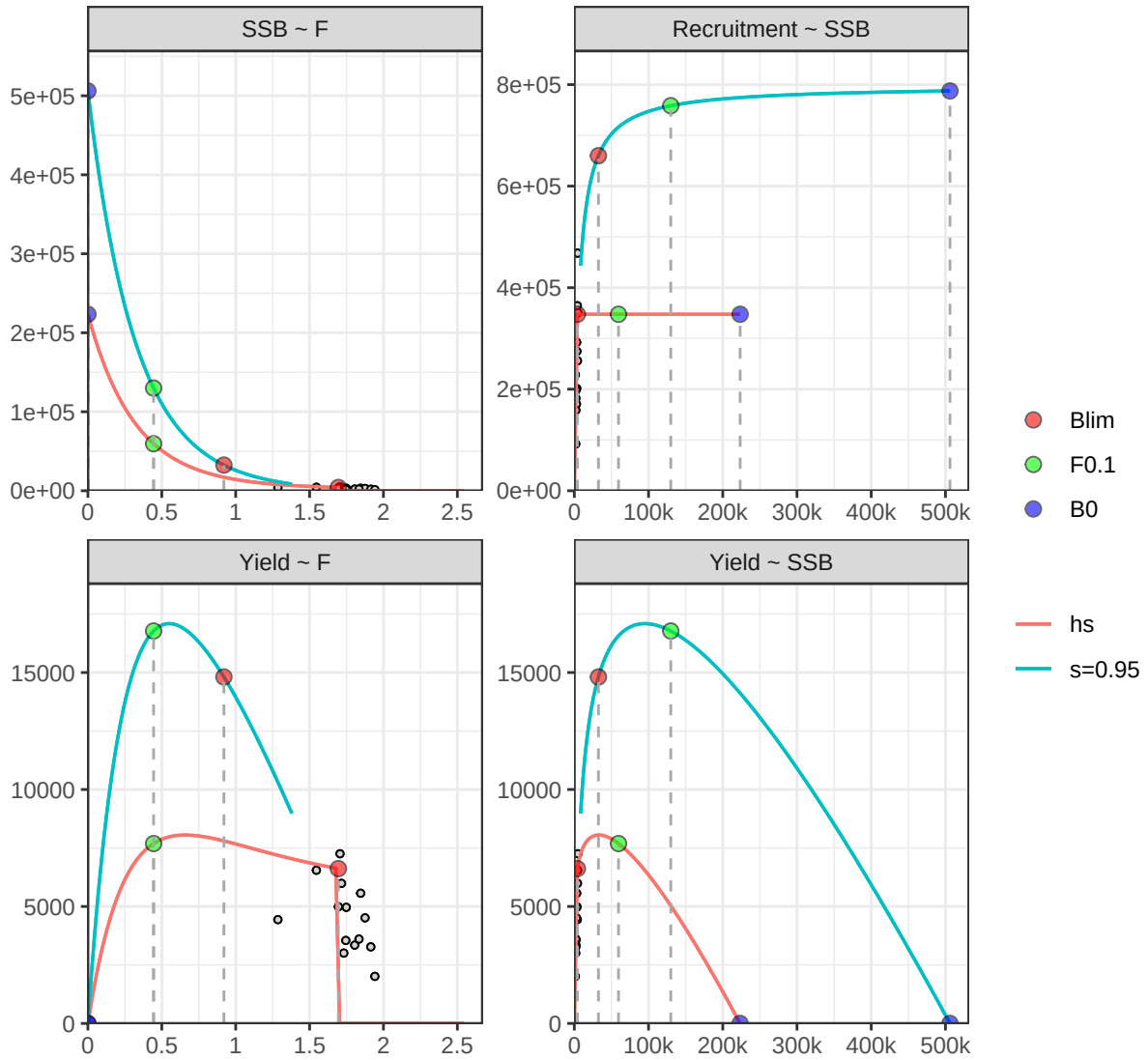


Figure 42: Reference point estimates of F_{01} , B_{lim} and B_0 shown as functions of SSB , F , Yield and Recruitment. Grey dots show the corresponding observations for HKE01.05.06.07

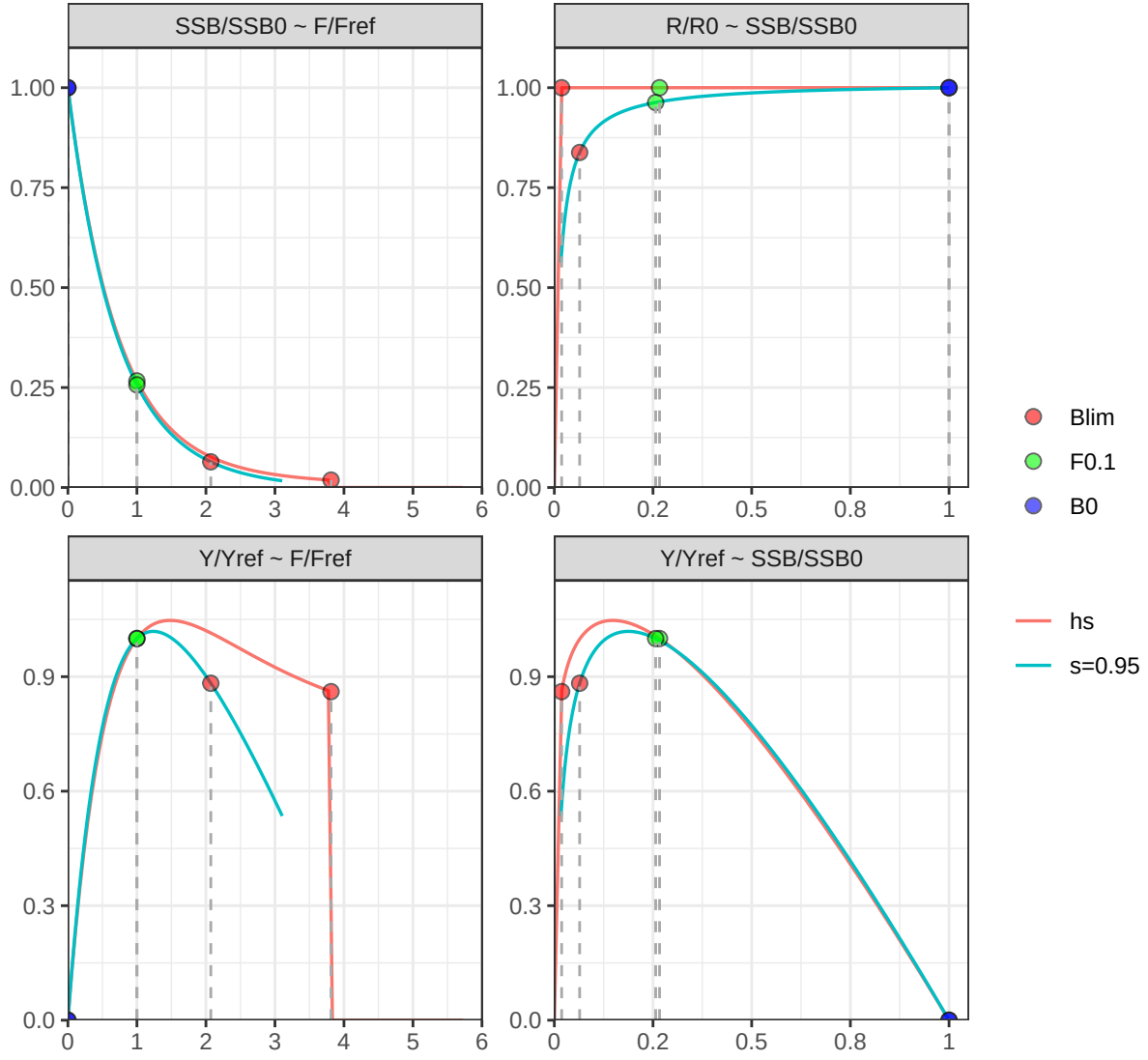


Figure 43: Relative reference point estimates of F_{01} , B_{lim} and B_0 shown as functions of SSB/SSB_0 , $F/F_{0.1}$, $Yield/Y_{F0.1}$ and R/R_0 . Grey dots show the relative values of the corresponding observations for HKE01.05.06.07

4.1.5 Implications for stock status of HKE01.05.06.07

```
plotAdvice(stk, brps$hs) + ggtitle(paste0(stk@name, ": Hockey-Stick"))
```

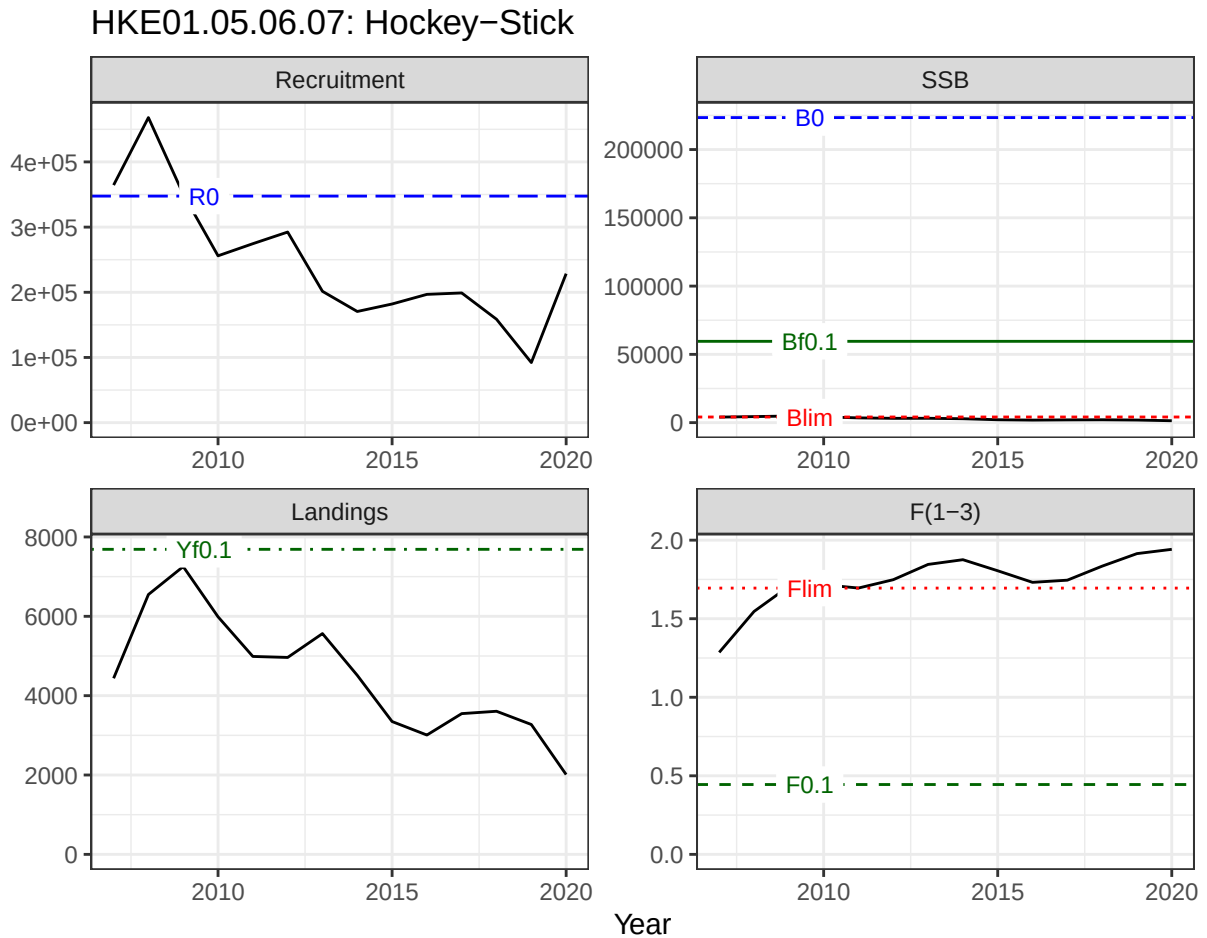


Figure 44: Status Advice plot for HKE01.05.06.07 showing stock trajectories of Recruitment, SSB , F , Landings and $Yield$ compared to the estimated reference points, based on a Hockey-Stick stock-recruitment relationship

```
plotAdvice(stk, brps[[2]]) + ggtitle(paste0(stk@name, ": Beverton-Holt"))
```

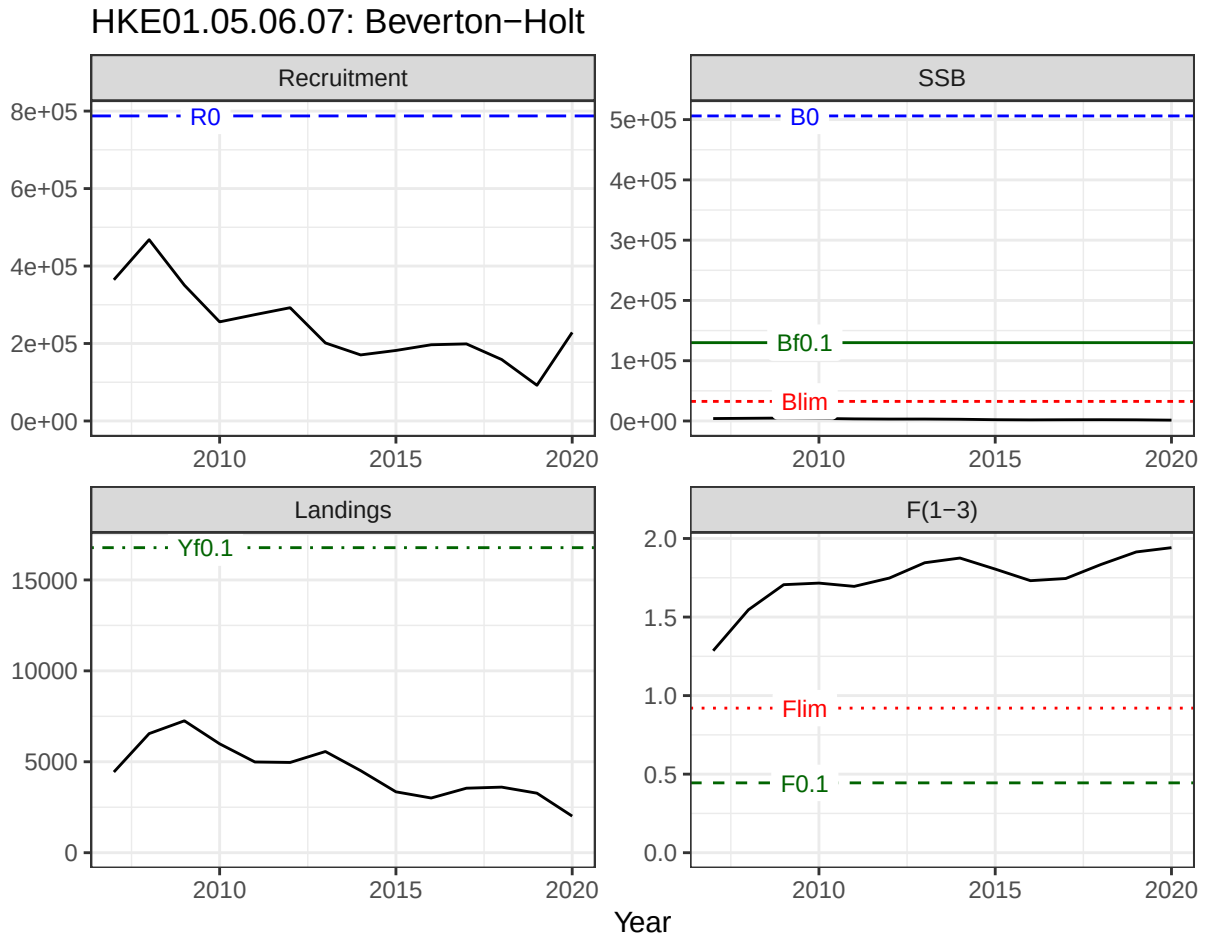


Figure 45: Status Advice plot for HKE01.05.06.07 showing stock trajectories of Recruitment, SSB , F , Landings and $Yield$ compared to the estimated reference points, based on a Beverton-Holt stock-recruitment relationship

```

p1 = plotAR(Fbrp(brps[[1]]), bpa = 1.4, kobe = FALSE, obs = stk) + ggtitle("a) Hockey-Stick (Absolute)")
p2 = plotAR(brps$hs, bpa = 1.4, rel = TRUE, kobe = FALSE, obs = stk) + ggtitle("b) Hockey-Stick (Relative)")
p3 = plotAR(brps[[2]], bpa = 1.4, kobe = FALSE, obs = stk) + ggtitle("c) Beverton-Holt (Absolute)")
p4 = plotAR(brps[[2]], bpa = 1.4, rel = TRUE, kobe = FALSE, obs = stk) + ggtitle("d) Beverton-Holt (Relative)")
ggarrange(p1, p2, p3, p4)

```

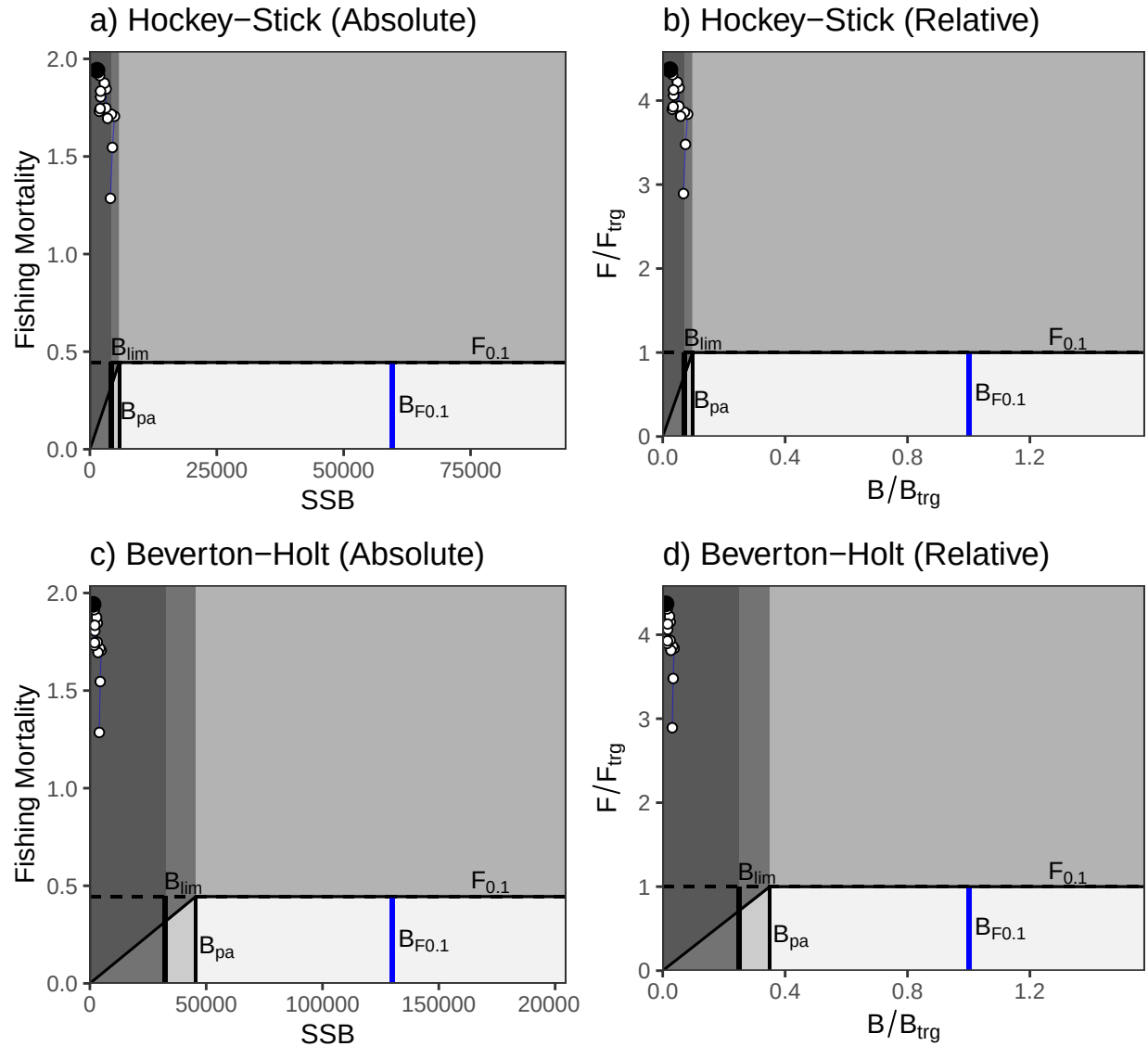


Figure 46: Advice Rule plots for HKE01.05.06.07, with $B_{pa} = 1.4B_{lim}$, showing a comparison between a Hockey-Stick and Beverton-Holt S-R with an empirical $B_{lim} = 0.25B_{F0.1}$.

4.2 HKE08.09.10.11

4.2.1 Stock dynamics of HKE08.09.10.11

Notable features of the HKE08.09.10.11 stock are:

- Compared to HKE01_05_06_07, notably older fish contribute meaningful to biomass of HKE08.09.10.11
- Selectivity is constant over time
- HKE08.09.10.11 is fully selected at age-1, after which selectivity decreases
- $F_{0.1}$ is relatively close to F_{max}
- YPR at $F_{0.1}$ is larger than YPR at the B_{lim} proxy of $0.25B_{F0.1}$
- The age-structure is severely truncated to unfished levels at $F = 0$, but notably less than for HKE01_05_06_07.
- – The corresponding spawning potential ratio less than $SRP = 0.10$ falls below the B_{lim} proxy.

```
plot(stk) + theme_bw() + facet_wrap(~qname, scales = "free", ncol = 2) + ggtitle(paste0(stk@name,
  " (", stk@species, ")"))
```

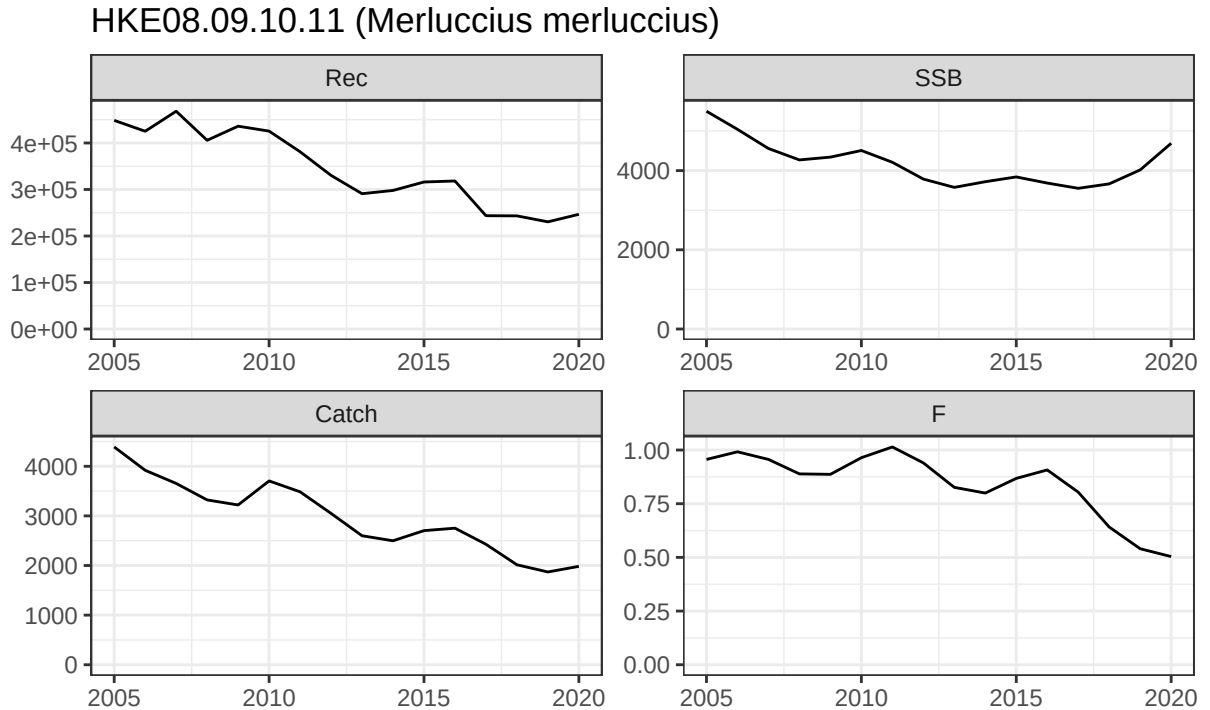


Figure 47: Estimated stock assessment trajectories for HKE08.09.10.11

```
plotdyn(stk) + ggtitle(paste0(stk$name, " (", stk$species, ")"))
```

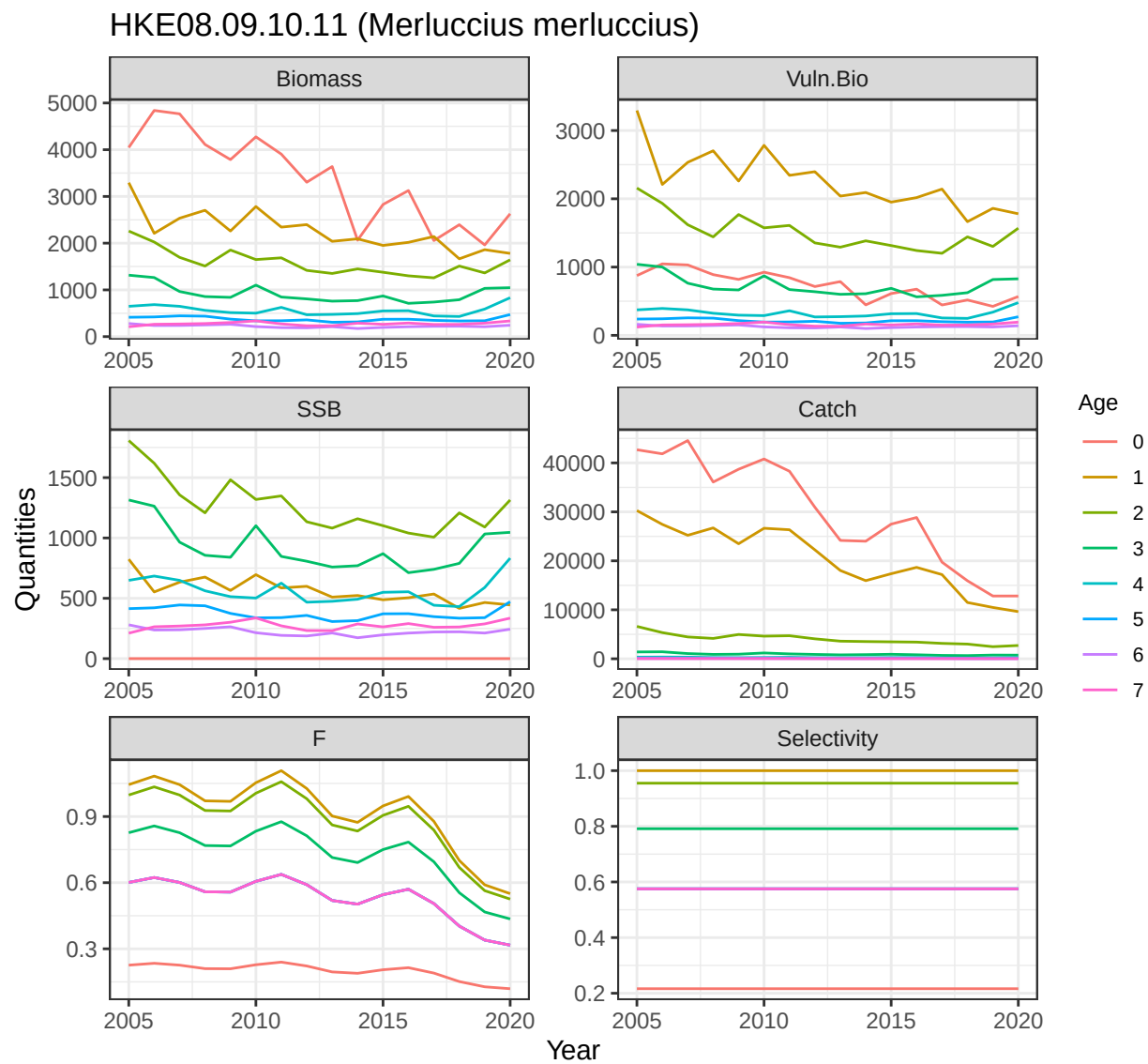


Figure 48: Stock assessment trajectories at age for HKE08.09.10.11

```
plotbiogr(stk) + ggtitle(paste0(stk@name, " (", stk@species, ")"))
```

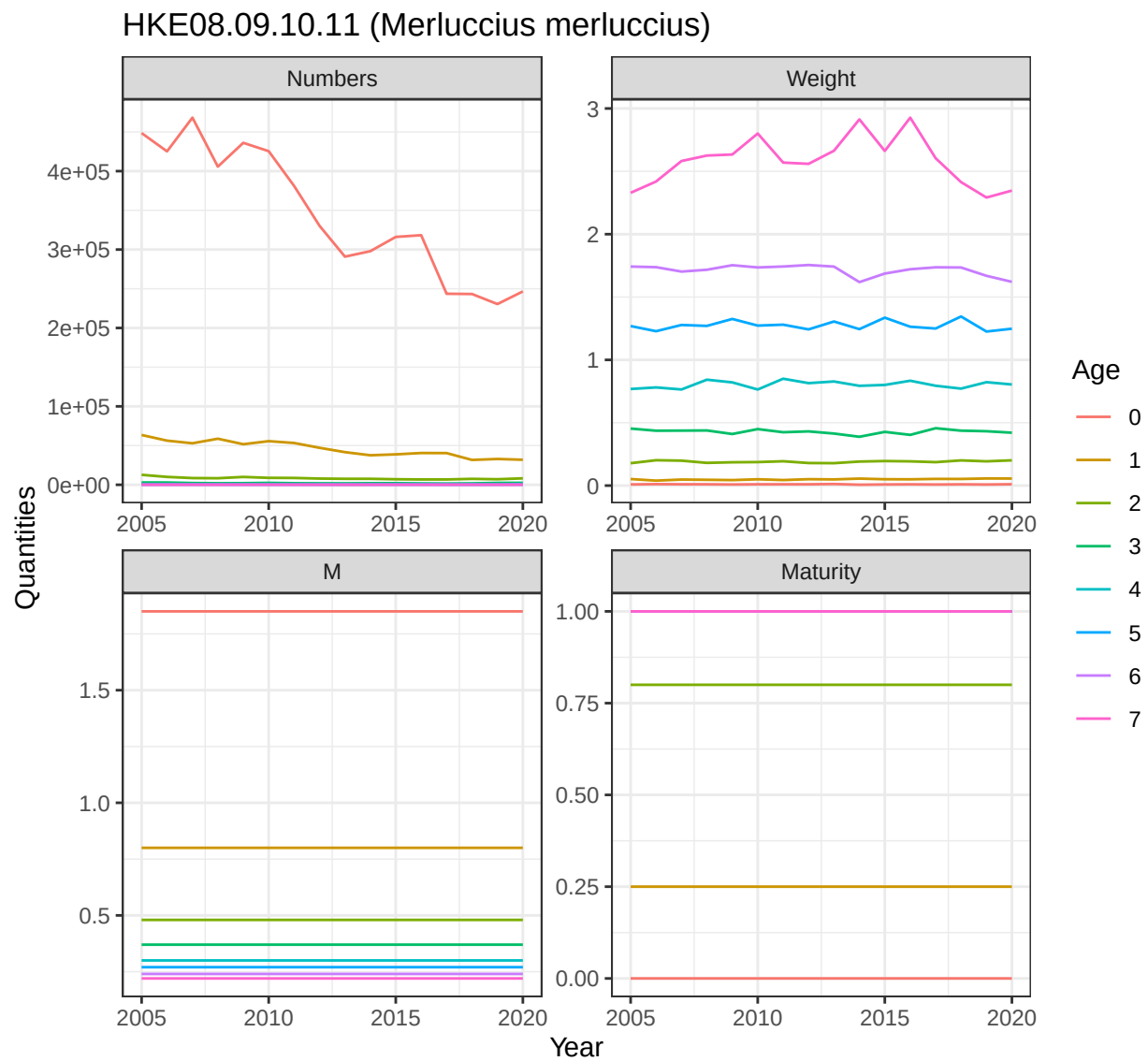


Figure 49: Stock biology trajectories at age for HKE08.09.10.11

```
plotbioage(stk) + ggtitle(paste0(stk@name, " (", stk@species, ")"))
```

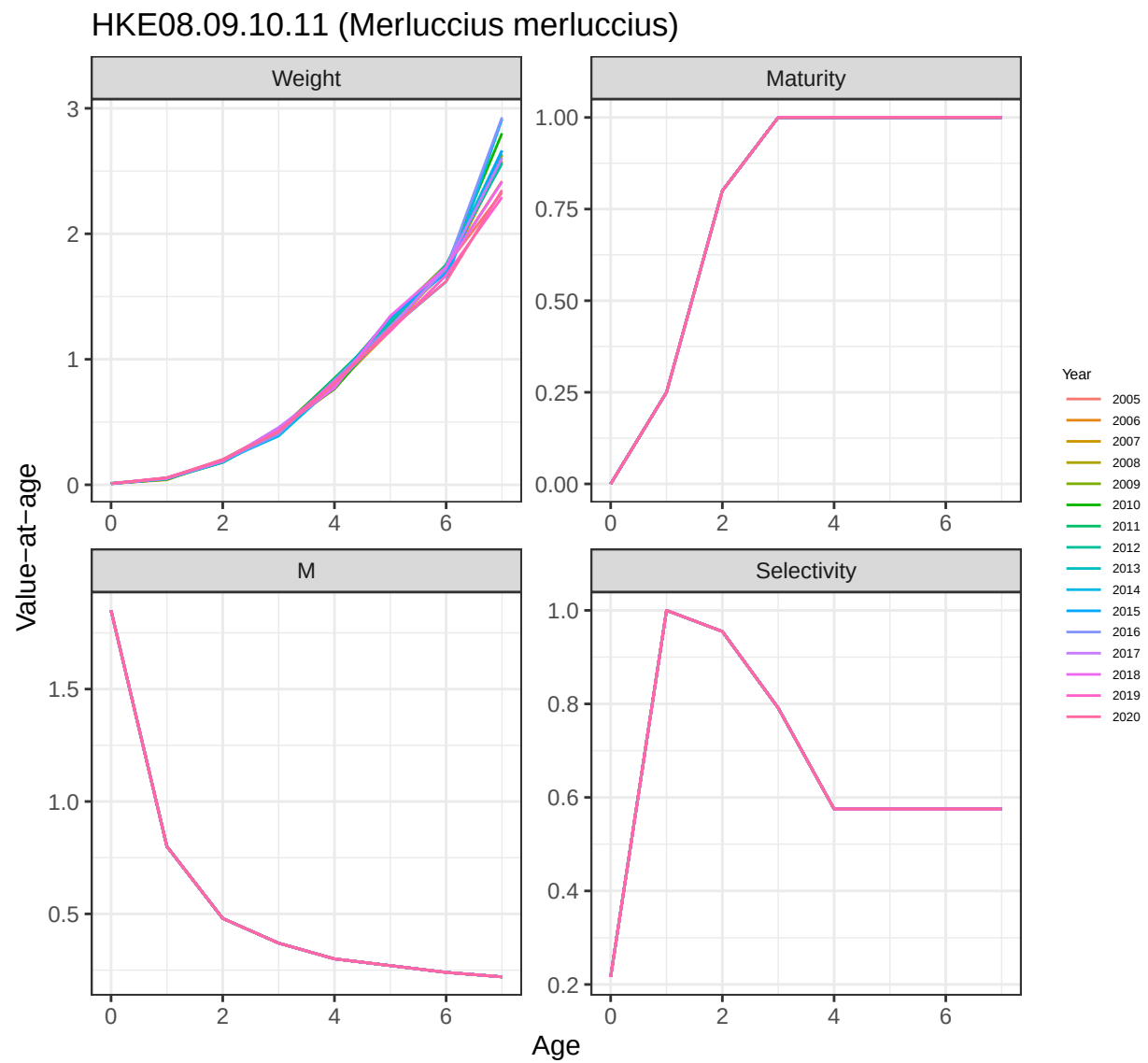


Figure 50: Annual stock quantities at ageHKE08.09.10.11

4.2.2 Per-recruit Analysis for HKE08.09.10.11

Compute the per-recruit reference points of interest

```
pr = computeFbrp(stk, proxy = "f0.1", blim = 0.25, type = "btrg", verbose = T)
Computing Per-Recruit Quantities
Computing F0.1 with Btrg = B[F.01]

Blim = 0.25 with Btrg corresponding to F0.1

# check
Fbrp(pr)
An object of class "FLPar"
params
  F0.1   Btrg   Blim   Flim   Yeq   B0
0.1680 0.1140 0.0285 0.5284 0.0117 0.2733
units:  NA
```

The following observations can be noted from the below results of the per-recruit analysis for HKE08.09.10.11:

- The YPR curve has distinct maximum, indicating that the stock is vulnerable to growth-overfishing
- Maximum yield per recruit (YPR) is attained as F approach infinity
- As a result YPR is higher at the B_{lim} proxy of $0.25B_{F0.1}$ (approx. $0.1SPR_0$) than at $F_{0.1}$
- The age-structure is severely truncated to unfished levels at $F = 0$, corresponding to spawning potential ratio of about $SRP = 0.1$

```
ploteq(pr) + ggtitle(paste0(stk@name, " (", stk@species, ")"))
```

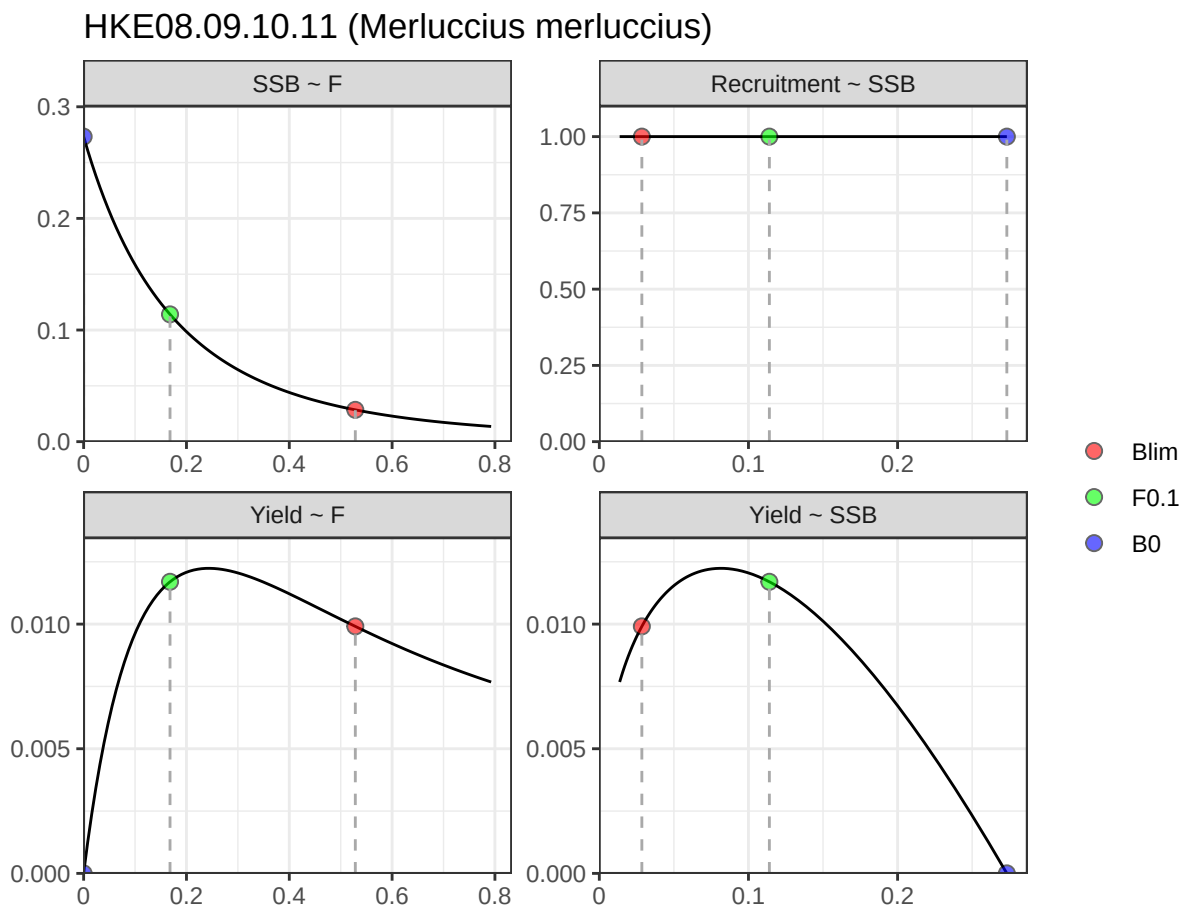


Figure 51: Per-recruit analysis for HKE08.09.10.11

```
plotspr(stk) + ggtitle(paste0(stk@name, " (", stk@species, ")"))
```

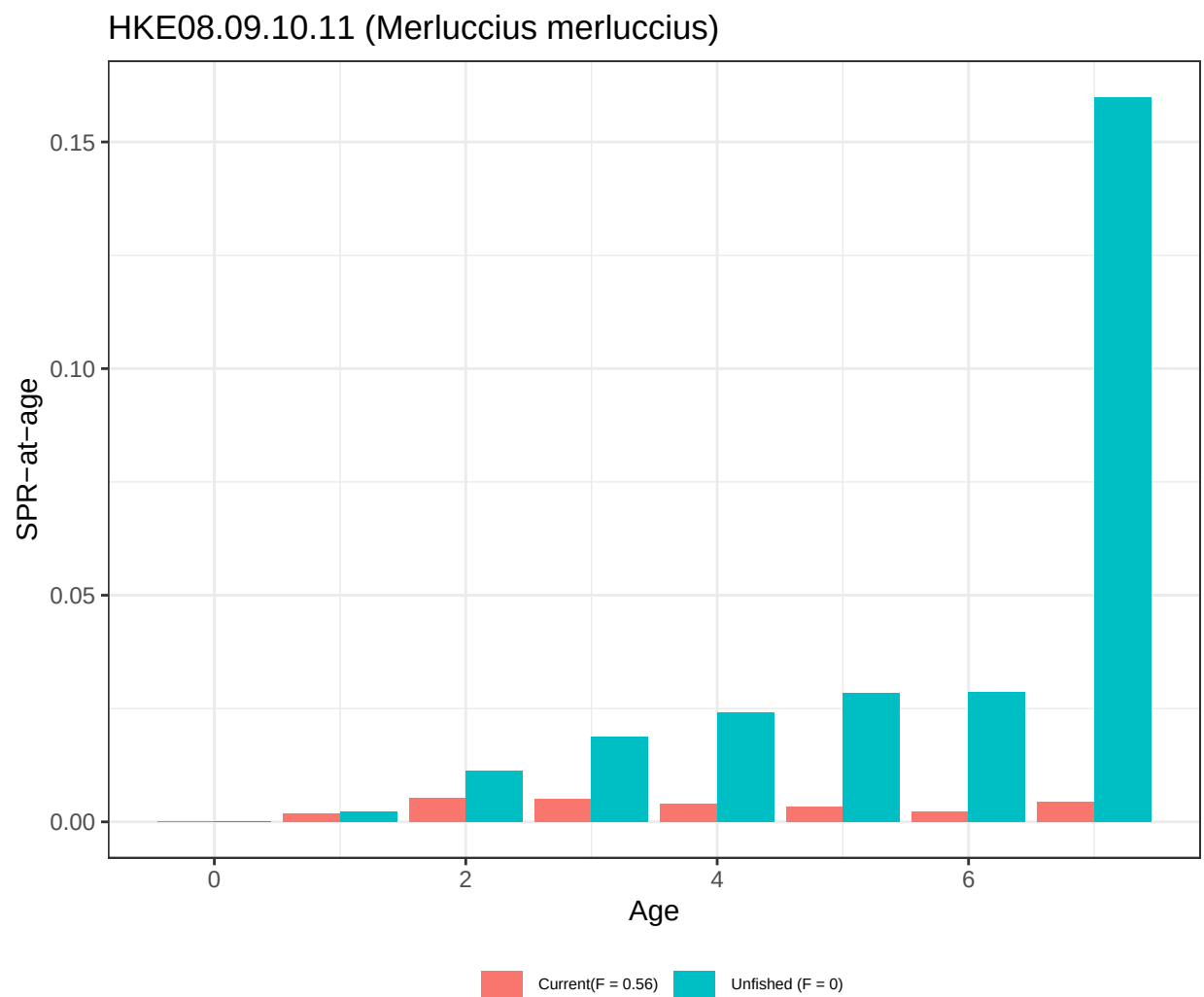


Figure 52: Comparison of the spawning biomass per recruit SPR_F at current F (average of last 3 years) and SPR_0 with $F = 0$ for HKE08.09.10.11

```
plotAdvice(stk, pr) + ggtitle(paste0(stk@name, " (", stk@species, ")"))
```

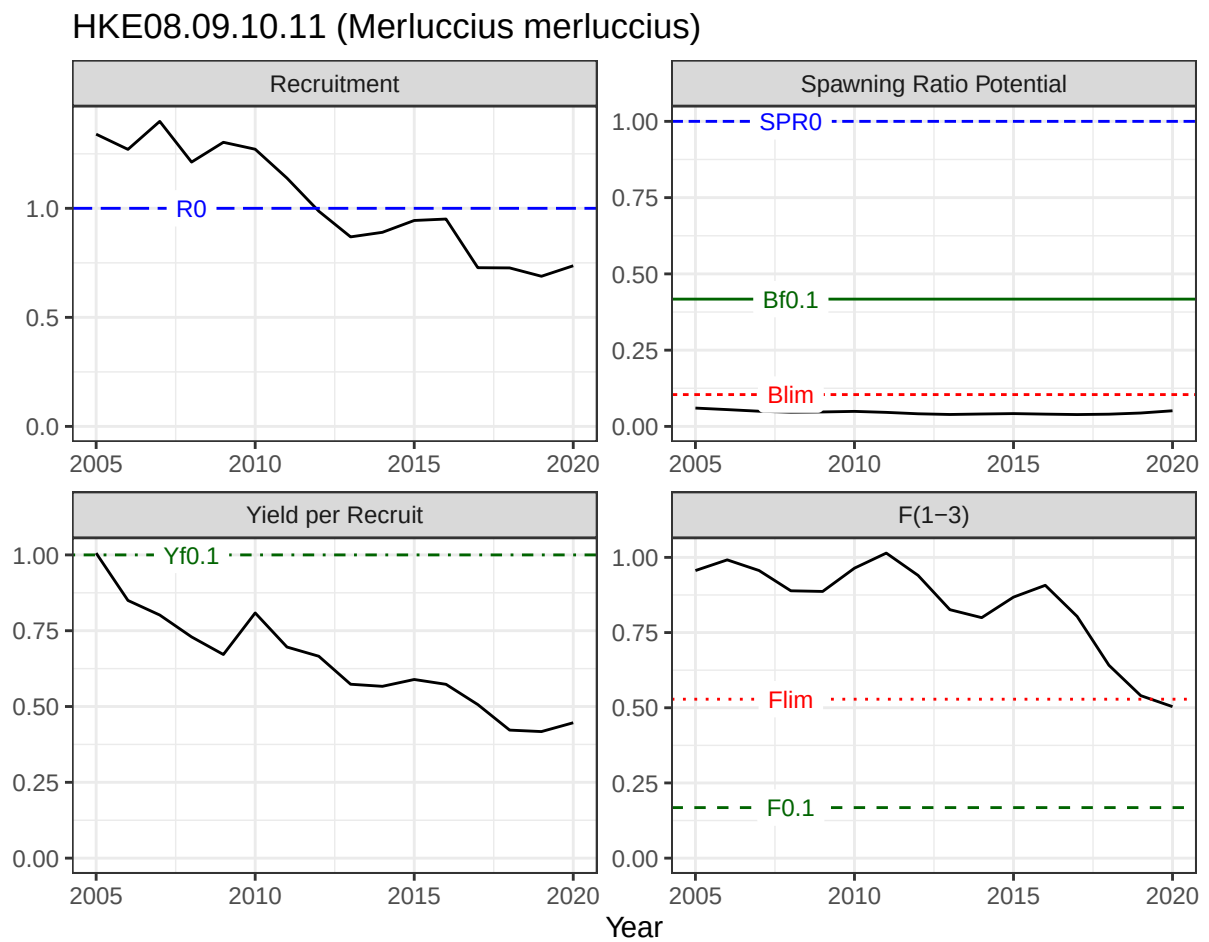


Figure 53: Time-series of stock quantities expressed as per-recruit HKE08.09.10.11

4.2.3 Fitting stock-recruitment functions for HKE08.09.10.11

Initially four recruitment functions are explored, using the function `srrTMB` in the package `FLSRTMB`:

1. Geometric Mean (`model=geomean`)
2. Hockey-Stick (`model=segreg`)
3. Beverton-Holt (`model=bevholtSV`)
4. Ricker (`model=ricker`)

The Hockey-Stick is constrained to search for solutions of the break-point so that it falls within a lower bound (`lplim`) and upper bound (`uplim`) of spawning ration potential $SRP_{lim} = SPR_{lim}/SPR_0$. Note that in the specific case of the Hockey-Stick $SRP_{lim} = B_{lim}/B_0$, but SRP_{lim} can be generalised even if the $b = B_{lim}$ estimate is inputted into another S-R. The initial bounds are chosen to be fairly on constrained for a range of $SRP_{lim} = SRP_{0.1-20}$ by setting `lplim=0.001` and `uplim=0.2`.

In the initial fits of the Beverton-Holt and Ricker models steepness s and $R0$ are estimated given the input SPR_{0_y} .

```
# compute annual spr0y
spr0 = spr0y(stk)
# Geometric Mean
gm = srrTMB(as.FLSR(stk, model = geomean), spr0 = spr0, nyears = 3)
# Constraint Hockey-Stick
hs = srrTMB(as.FLSR(stk, model = segreg), spr0 = spr0, lplim = 0.001, uplim = 0.2,
  nyears = 3)
# Beverton Holt
bh = srrTMB(as.FLSR(stk, model = bevholtSV), spr0 = spr0, nyears = 3)
# Ricker
ri = srrTMB(as.FLSR(stk, model = rickerSV), spr0 = spr0, nyears = 3)

# Estimates
gm@SV
  s      sigmaR      R0      rho      B0
1 NA 0.2465352 334757.6 0.9102787 91479.74
hs@SV
  s      sigmaR      R0      rho      B0
1 NA 0.2073914 379350.9 0.8784284 103665.8
bh@SV
  s      sigmaR      R0      rho      B0
1 0.8850482 0.2092284 1342544 0.8658003 366879.2
ri@SV
  s      sigmaR      R0      rho      B0
1 2.489579 0.1849167 773625.3 0.8783716 211409.8

# Combine in one object
srs = FLSRs(gm = gm, hs = hs, bh = bh, ri = ri)
```

```

p1 = plotsrs(srs)
p2 = plotsrts(srs)
p3 = plotsrs(srs, b0 = T)
p4 = plotsrs(srs, b0 = T, rel = T)

```

```

ggarrange(p1, p2, p3, p4, ncol = 2, nrow = 2, common.legend = TRUE, legend = "right")

```

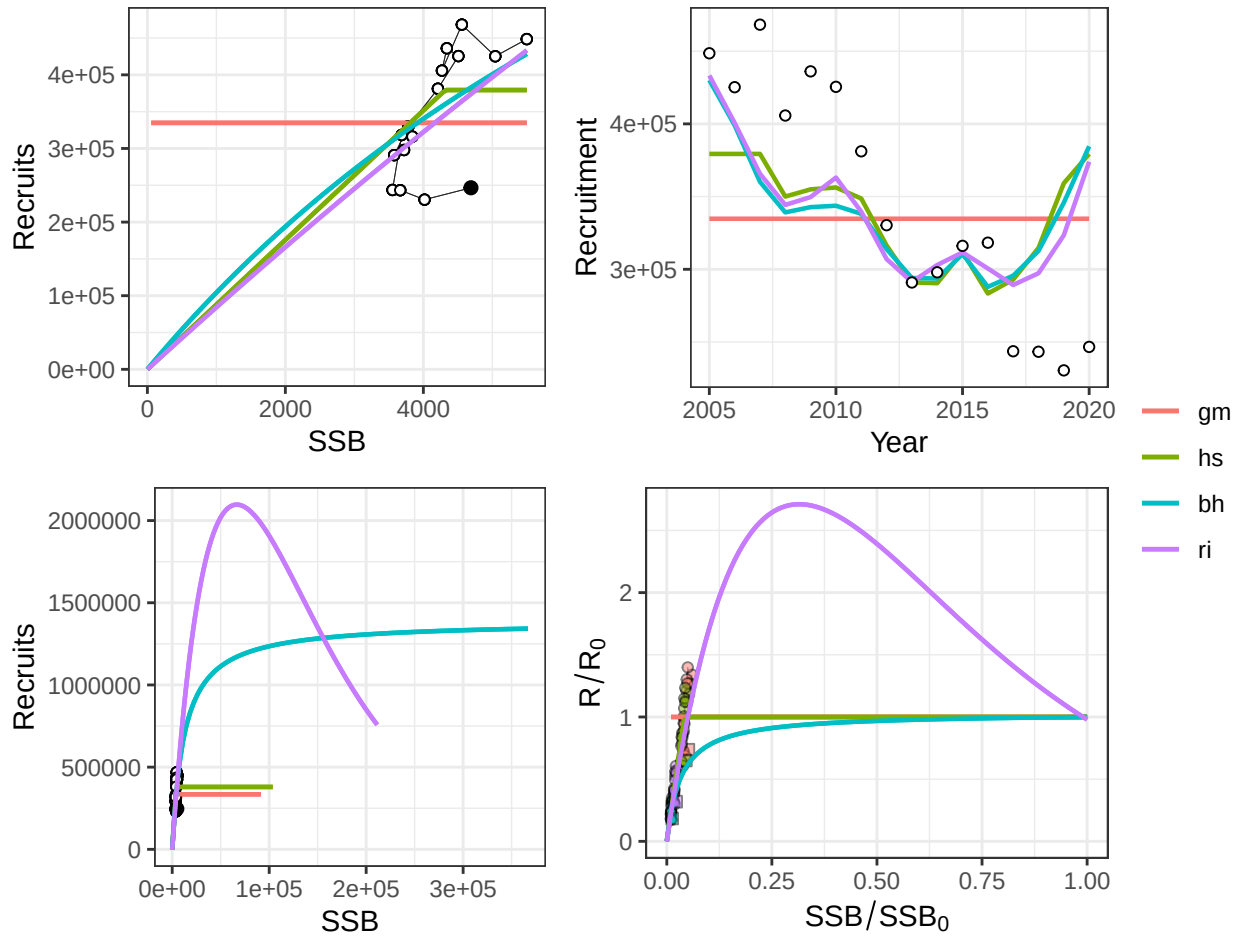


Figure 54: Fits of alternative stock-recruitment relationship for HKE08.09.10.11 assuming a geometric mean (gm), conditioned hockey-stick (hs), beverton-holt (bh) and ricker (ri) model

The results show that the recruitment variation is fairly low, e.g. $\sigma_r = 0.21$ for the Beverton-Holt model. The predicted recruitment by Hockey-Stick, Beverton-Holt and Ricker models follow the observed recruitment pattern over time, however, the three models differ largely in scale of their R_0 and B_0 estimates.

The break-point of the hockey-stick is estimated at $b = 4316$ corresponding to an $SRP_{lim} = 0.042$ and the corresponding R_0 is slightly above the geometric mean recruitment.

In the following the sensitivity to alternative fixed steepness values of $s = 0.8 - 0.95$ for the Beverton-Holt model is explored. The results show that increasing s to 0.8-0.85 substantially decrease the R_0 and B_0 estimates to a scale that is comparable to the Hockey-Stick estimates.

```
# range of s
s = seq(0.8, 0.95, 0.05)
bhs = FLSRs(lapply(s, function(x) {
  srrTMB(as.FLSR(stk, model = bevholtSV), spr0 = spr0y(stk), s.est = FALSE, s = x)
}))

# require unique names
names(bhs) = paste0("s=", s)

# Add Geomean and Hockey-Stick SRR
srs[1:2]@names
[1] "gm" "hs"

bhs = FLSRs(c(srs[1:2], bhs))
```

```
p1 = plotsrs(bhs, b0 = F)
p2 = plotsrts(bhs)
p3 = plotsrs(bhs, b0 = T)
p4 = plotsrs(bhs, b0 = T, rel = T)
ggarrange(p1, p2, p3, p4, ncol = 2, nrow = 2, common.legend = TRUE, legend = "right")
```

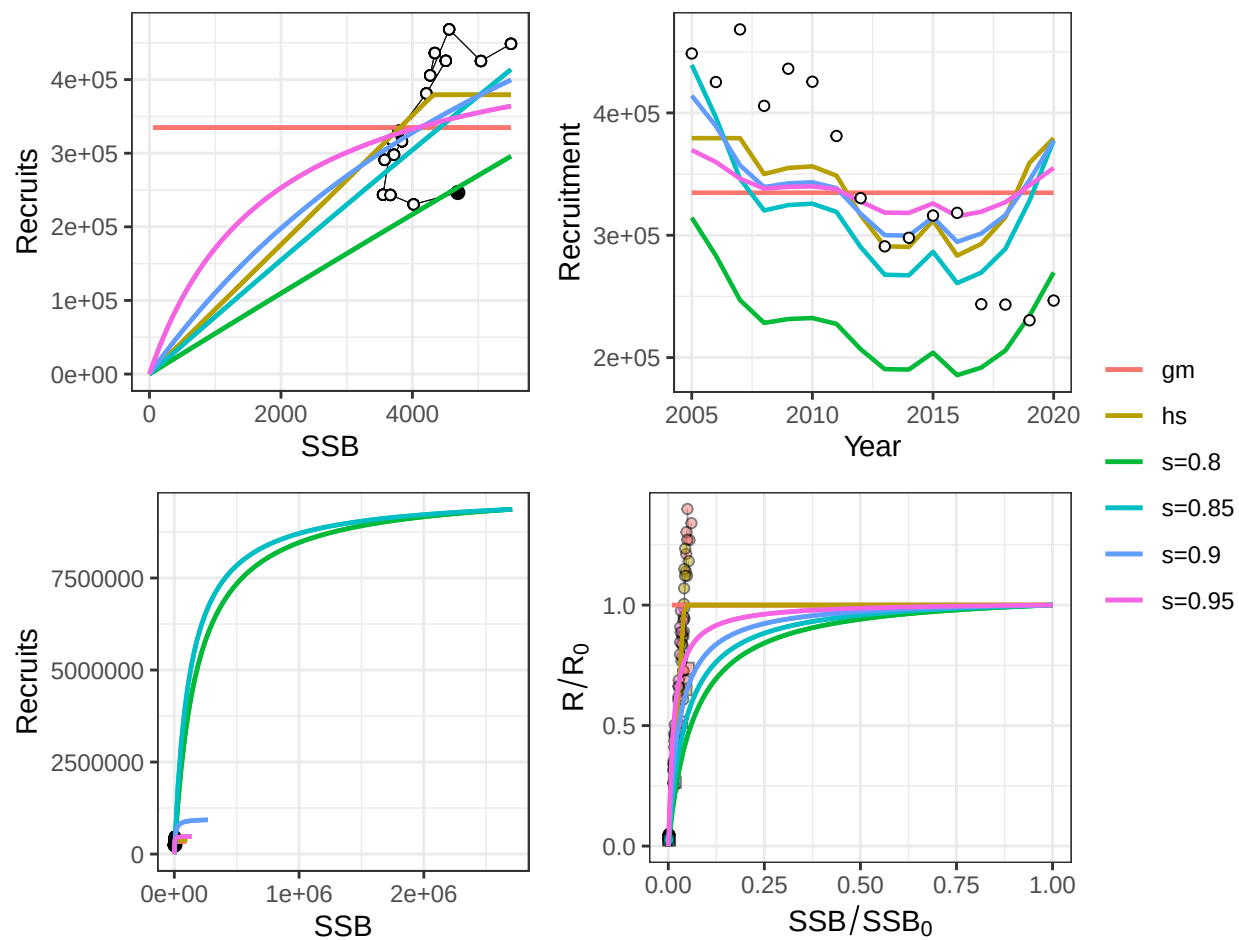


Figure 55: Fits of stock-recruitment relationships for HKE08.09.10.11 assuming a geometric mean (gm), conditioned hockey-stick (hs), alternative steepness s values for beverton-holt (bh) model

4.2.4 Reference point estimation for HKE08.09.10.11

The following candidate S-R models are considered for initial reference point estimation:

1. Hockey-Stick with B_{lim} set to the break-point $b = 4316$
2. Beverton-Holt with a fixed $s = 0.95$ with $B_{lim} = 0.25B_{F0.1}$ ⁴

```
# Select S-R models
selsr = FLSRs(c(srs[2], bhs[6]))
# Extract blim
blim = an(hsblim(hs)["Blim"])
# set up varying Blim parameterisation
blims = c(blim, 0.25)
types = c("value", "btrg")

brps = FLBRPs(Map(function(x, y, z) {
  computeFbrp(stk, x, proxy = "f0.1", blim = y, type = z, verbose = FALSE, fmax = 10)
}, x = selsr, y = blims, z = types))
```

```
ploteq(brps, obs = T)
```

```
ploteq(brps, obs = T, rel = T)
```

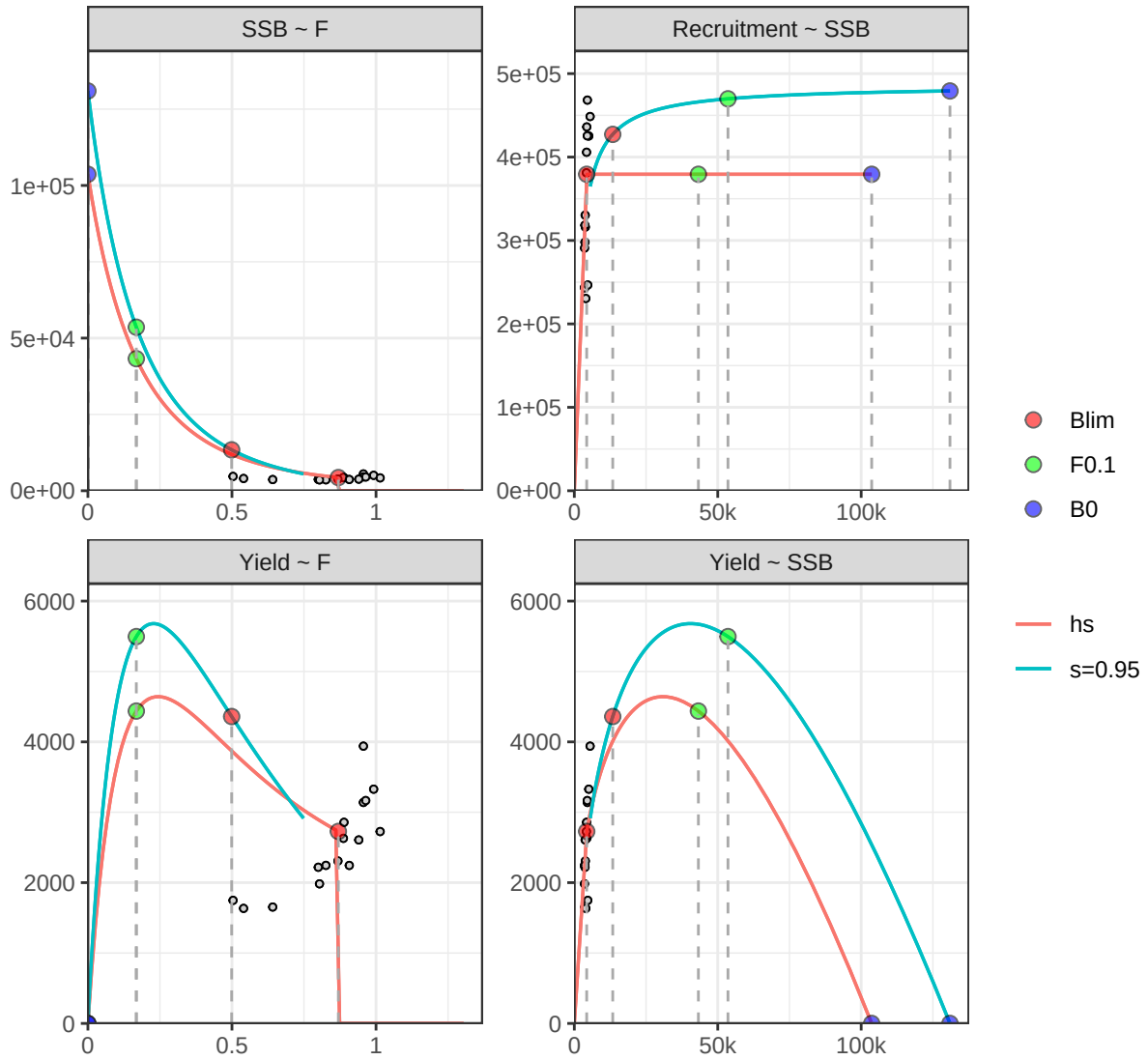


Figure 56: Reference point estimates of F_{01} , B_{lim} and B_0 shown as functions of SSB , F , Yield and Recruitment. Grey dots show the corresponding observations for HKE08.09.10.11

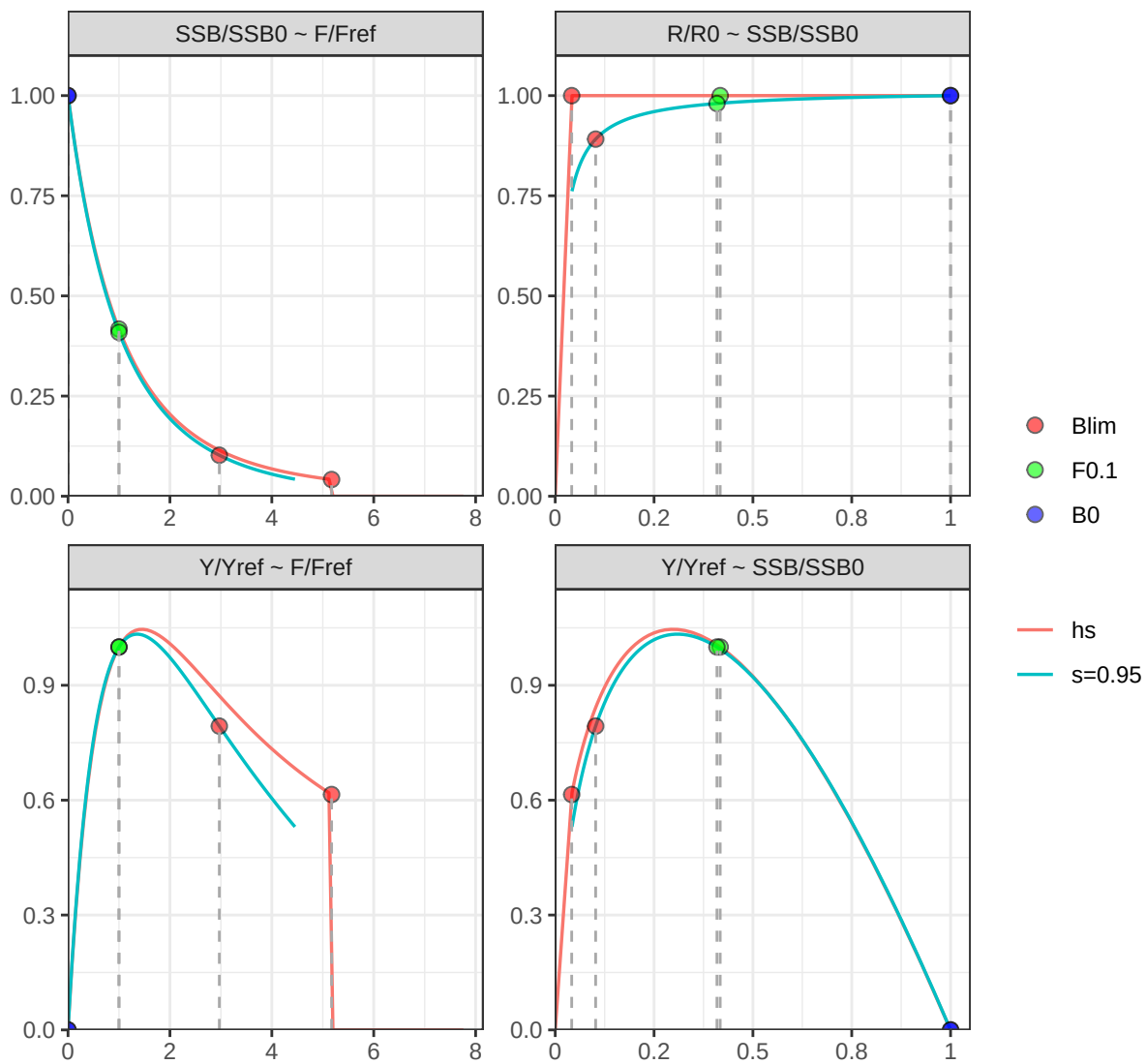


Figure 57: Relative reference point estimates of F_{01} , B_{lim} and B_0 shown as functions of SSB/SSB_0 , $F/F_{0.1}$, $Yield/Y_{F0.1}$ and R/R_0 . Grey dots show the relative values of the corresponding observations for HKE08.09.10.11

4.2.5 Implications for stock status of HKE08.09.10.11

```
plotAdvice(stk, brps$hs) + ggtitle(paste0(stk@name, ": Hockey-Stick"))
```

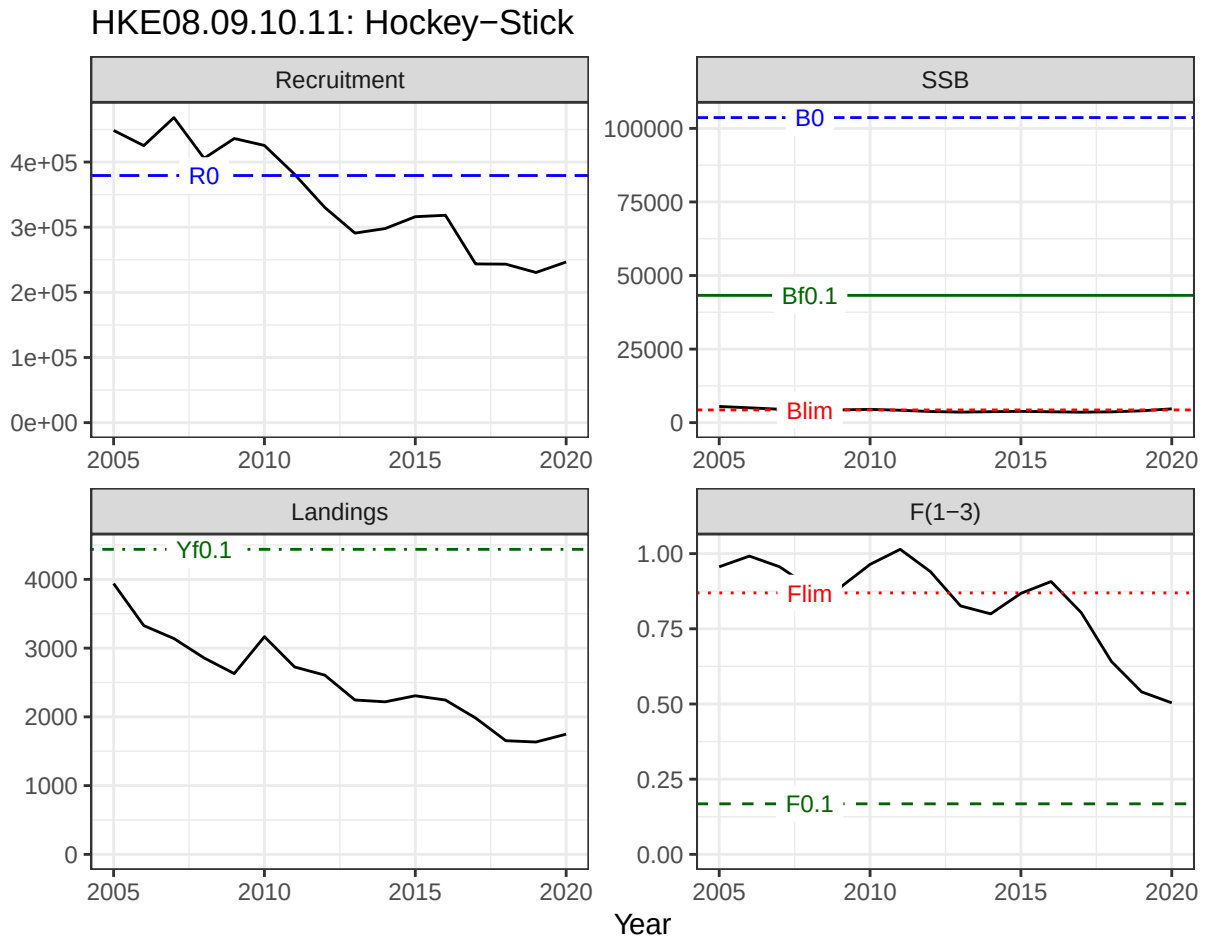


Figure 58: Status Advice plot for HKE08.09.10.11 showing stock trajectories of Recruitment, SSB , F , Landings and $Yield$ compared to the estimated reference points, based on a Hockey-Stick stock-recruitment relationship

```
plotAdvice(stk, brps[[2]]) + ggtitle(paste0(stk@name, ":", "Beverton-Holt"))
```

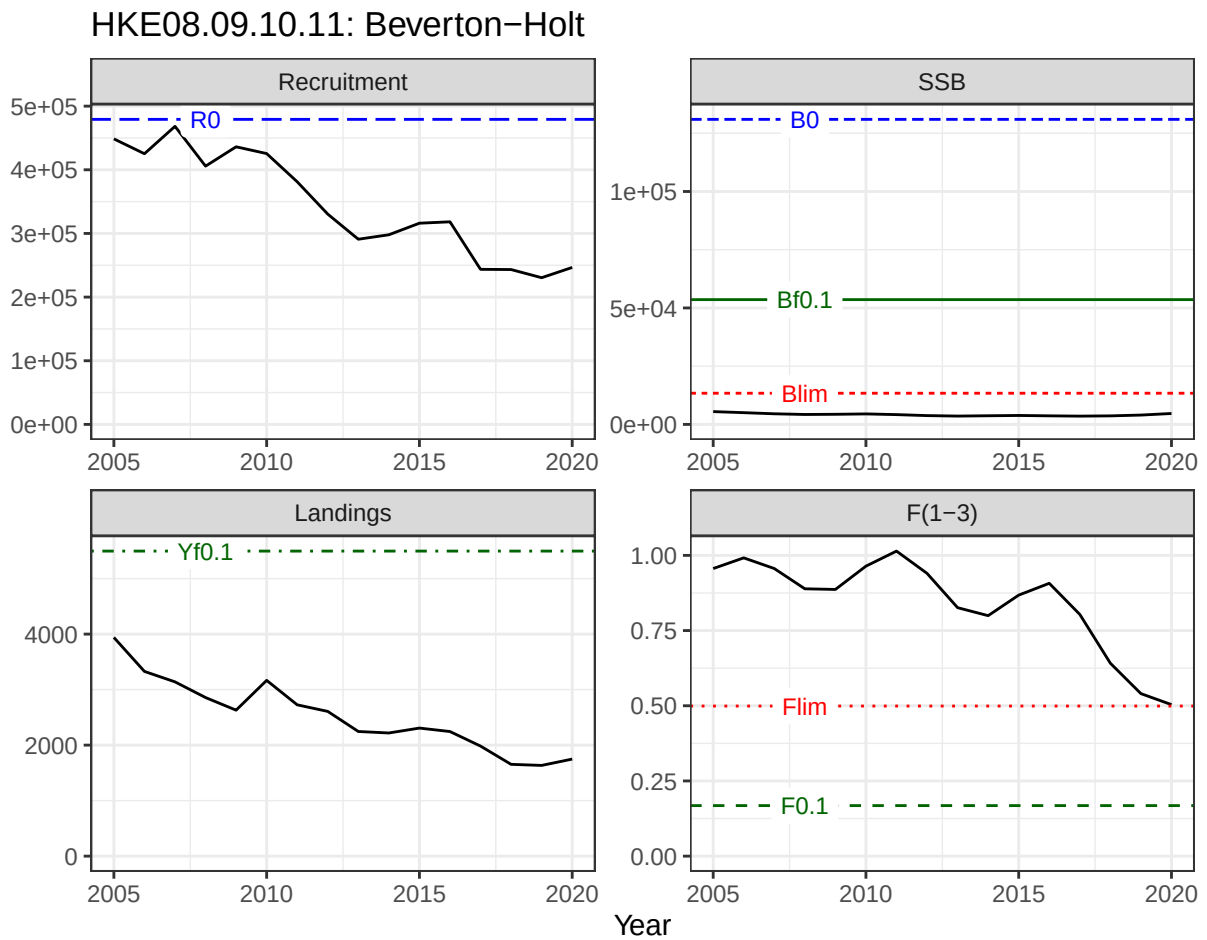


Figure 59: Status Advice plot for HKE08.09.10.11 showing stock trajectories of Recruitment, SSB , F , Landings and $Yield$ compared to the estimated reference points, based on a Beverton-Holt stock-recruitment relationship

```

p1 = plotAR(Fbrp(brps[[1]]), bpa = 1.4, kobe = FALSE, obs = stk) + ggtitle("a) Hockey-Stick (Absolute)")
p2 = plotAR(brps$hs, bpa = 1.4, rel = TRUE, kobe = FALSE, obs = stk) + ggtitle("b) Hockey-Stick (Relative)")
p3 = plotAR(brps[[2]], bpa = 1.4, kobe = FALSE, obs = stk) + ggtitle("c) Beverton-Holt (Absolute)")
p4 = plotAR(brps[[2]], bpa = 1.4, rel = TRUE, kobe = FALSE, obs = stk) + ggtitle("d) Beverton-Holt (Relative)")
ggarrange(p1, p2, p3, p4)

```

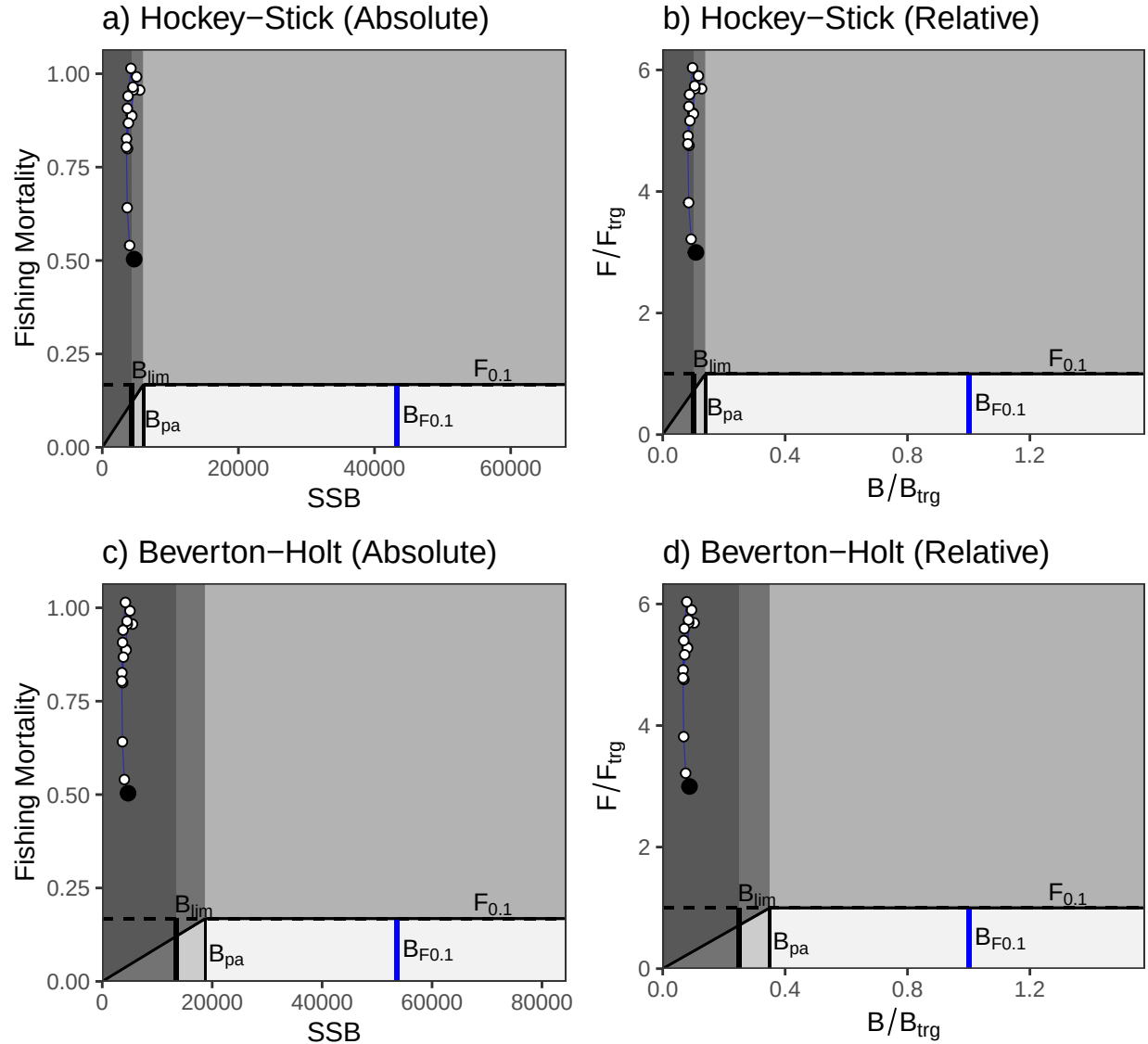


Figure 60: Advice Rule plots for HKE08.09.10.11, with $B_{pa} = 1.4B_{lim}$, showing a comparison between a Hockey-Stick and Beverton-Holt S-R with an empirical $B_{lim} = 0.25B_{F0.1}$.